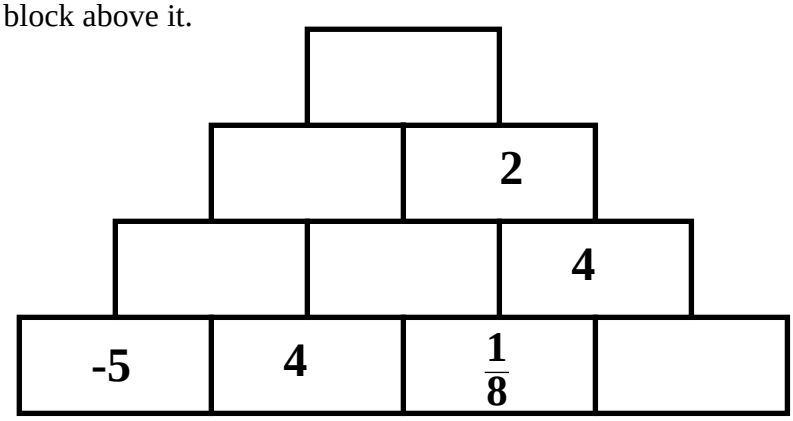
MathStars

a problem solving newsletter

Vol. 8 No. 1

★★ 1. Simplify: $$\frac{\frac{5}{3}+\frac{5}{3}+\frac{5}{3}}{\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}}$$

★★★ 2. In the pyramid some blocks contain numbers. To fill in the empty blocks multiply the numbers in adjacent blocks to fill in the block above it.



★ 3. Quincy has three pairs of shorts and five coordinating shirts. How many days can Quincy wear a different combination of shirts and shorts?_____

★★★★ 4. Find the next three numbers in this sequence by following the directions:

To find the term after n, divide (n-1) by n.

12, ____, ____, ____

What number came before 12? _____

★★★★ 5. Otto asked Bob, " How many three-digit numbers are palindromes?" How should Bob answer the question?

_____

Strategy of the Month

Someone said, "A picture is worth a thousand words." Turning the words of a problem into a picture or a diagram can help you "see" the problem. By using the part of your brain that visualizes a situation or object, you may see relationships or information that helps you solve the problem. When someone tells you a story, try turning the words into a motion picture or a cartoon. When reading a description, try "seeing it in your mind's eye." If you can do these things, this strategy may be for you! Try using a picture or make a diagram to solve this problem:

Ten people met at a party. They all exchanged handshakes. How many handshakes were exchanged?

MathStars Home Hints

Every year you grow and change in many different ways. Get someone to help you measure and record these data about yourself. Be sure to save the information because we will measure again in two months!

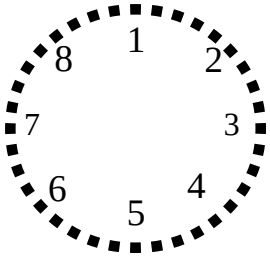
How tall are you? _____

How much do you weigh? _____

What is the circumference of your head?

_____

★★★ 6. The numbers on the "clock" below make up a clock number system. This is called the modulo-8 or mod 8 system. If you start at 8 and move clockwise, then 10 is really 2 in the mod 8 system. 15 = 7 (mod 8)



a) Find the mod 8 equivalent of the following numbers:

27 = _____ (mod 8)

144 = _____ (mod 8)

-18 = _____ (mod 8)

b) Two numbers that have the same modulo equivalence are called congruent. Explain why 14 is congruent to 30 (mod 8).

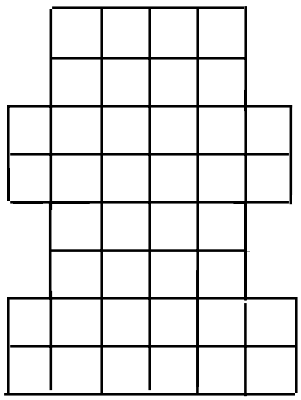
c) Draw a clock for a modulo-12 system and solve the following:

56 = _____ (mod 12)

16 = _____ (mod 12)

-34 = _____ (mod 12)

★★ 7. In the figure below each small square has an area of one square centimeter. The shape can wrap, with no overlap, a rectangular solid.



a) Find the volume of the rectangular solid that can be wrapped. _____

b) Find the surface area of the solid that can be wrapped. _____

Setting Personal Goals

Problem solving is what you do when you don't know what to do. Being a good problem solver will help you be ready to live and work in our changing world. Computers can do computations but people must tell the computers what to do. Good problem solvers know how to make plans and use many different strategies in carrying out their plans. They use all of their past experiences to help them in new situations. We learn to swim by getting in the water; we learn to be good problem solvers by solving problems!

MathStars
Commentary for Teachers
Vol. 8 No. 1

About these newsletters...

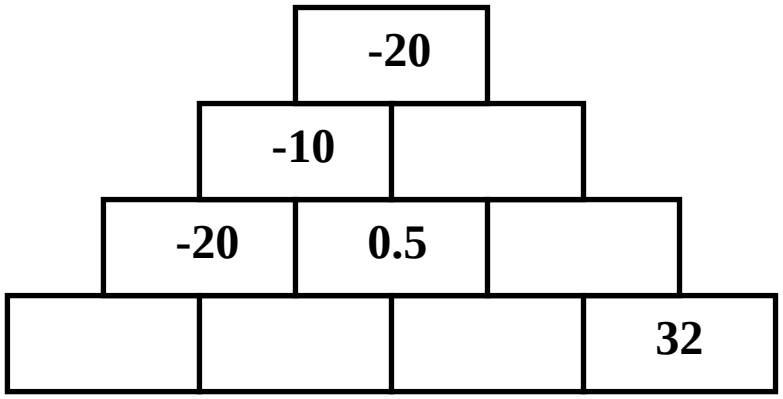
The purpose of the MathStars Newsletters is to challenge students beyond the classroom setting. Good problems can inspire curiosity about number relationships and geometric properties. It is hoped that in accepting the challenge of mathematical problem solving, students, their parents, and their teachers will be led to explore new mathematical horizons.

As with all good problems, the solutions and strategies suggested are merely a sample of what you and your students may discover. Enjoy!!

Discussion of the problems...

1. **(5/3)**
$$\frac{\frac{15}{3}}{\frac{15}{5}}$$

2.



3. **(15 outfits)** The n x m rule applies here, i.e. 3 x 5 = 15.

4. **(11/12, -11, 12/11; -1/11)** The number before 12 can be described by (x-1)/x =12.

5. **(90)** A three-digit palandrome can be a number like 444 or 434. The first and third digits can be any number one through nine. The middle digit can be any of ten digits zero through nine. Using the n x m rule there are 9 x 10 or 90 possibilities.

6. **(a) 3; 8; 2 b) both are conguent or equivalent to 6 (mod 8) c) 6; 4; 2)** Students may note that if a = b (mod c) then b is the remainder when a is divided by c.

7. **(a) 16 cubic centimeters; b) 40 square centimeters)** The shape can cover a solid with six faces: four 2 x 4's and two 2 x 2's. The solid is therefore a 2 x 4 x 2 parallelepiped. Students can cut, fold and wrap to model the solid.

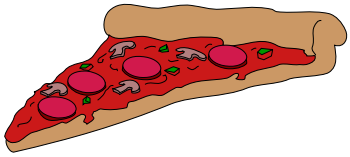
MathStars

a problem solving newsletter

Vol. 8 No. 2

★★ 1. Find all the positive integers less than 1000 that are divisible by 2, 3, 5, and 7.

★★ 2. An eight inch diameter pizza costs $3.85. Kate asserts that a 16 inch diameter pizza (same toppings, same crust) should cost $7.70. Explain her reasoning, and determine if this is the fair price. What cost would you propose as fair? Why?



★★★★ 3. The figure below has four rows; the first row contains a one.

```
                1
            2       3
        4       5       6
    7       8       9       10
```

a) What are the numbers in the 12th row?

b) What is the sum of the numbers in the 72nd row?

★★ 4. Arrange the digits 1, 3, 4, 6, 9 in the spaces below to give the largest possible product.

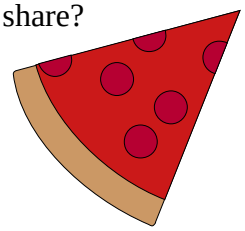
_____ _____ _____

X _____ _____

_____

★ 5. There are 44 boys in the eighth grade class of Gordon Middle School. There are 14 boys who do not play football and six of the football players do not play basketball. How many eighth grade boys play both football and basketball?

★★★ 6. Johna and her three friends order the same thing for lunch except Johna has an extra slice of pizza for $1.25. If the total bill is $10.45, what is Johna's share?



Strategy of the Month

Your brain is an organizer. It organizes information as it stores that information. When a problem involves many pieces of information, your brain will have an easier time sorting through it if you make an organized list. A list helps you be sure you have thought of all of the possibilities without repeating any of them. Like drawing a picture or making a diagram, making an organized list helps your brain "see" the problem clearly and find a solution. Try ***making an organized list*** *to solve this problem:*

You receive a penny on January 1st, two cents the next day, four cents the next and so on, doubling every day. How long will it take to be a millionaire? Suppose you started with a nickel and doubled, how long before you are worth a million dollars?

MathStars Home Hints

Sometimes the hardest part of solving a problem is just getting started. Having some steps to follow may help you.

1. Understand the information in the problem and what you are trying to find out.

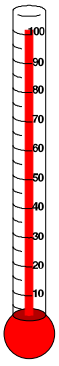
2. Try a strategy you think might help you solve the problem.

3. Find the solution using that strategy or try another way until you solve the problem.

4. Check back to make certain your answer makes sense.

★ 7. A box in a dark closet contains seven green balls, five red balls, and four black balls. How many balls must you take to be sure you have at least two balls of every color?

★★ 8. Scientists are studying the temperature changes at an experimental weather station. The first reading is 22˚ F. If the temperature rose 10˚ at the second reading, dropped 25˚ at the third reading, dropped 15˚ at the fourth, and rose 21˚ at the fifth. When was the lowest reading recorded?



★★★ 9. Some new operations are defined as follows:

$$a \# b = a \times b + 2$$

i.e. $3 \# 5 = 3 \times 5 + 2 = 17$, and

$$a \,\&\, b = a + b \times 2$$

i.e. $3 \,\&\, 5 = 3 + 5 \times 2 = 13$.

Find the value of:

a) $12 \# 3$

b) $16 \,\&\, 7$

c) $(3 \,\&\, 10) \# 12$

Is either operation commutative? How do you know?

Setting Personal Goals

Being able to ask good questions will help you in many ways. Use these to solve problems:

- *What information do I know?*
- *What else do I need to find out?*
- *What question am I trying to answer?*
- *Have I missed anything?*
- *Does my answer make sense?*

Set the goal of asking good questions!

MathStars
Commentary for Teachers
Vol. 8 No. 2

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Discussion of the problems...

1. **(210, 420, 630)** The numbers must be multiples of 2 x 3 x 5 x 7 or 210.

2. **(She thinks that double the diameter gives twice as much pizza, and therefore the price should be double. Actually the area is four times greater so the price should be close to $15.40)** Comparing areas yields the following ratio: eight inch pizza : 16 inch pizza or 1 : 4 i. e. Area = πr^2 or 3.14 x 16 ≈ 50 square inches while 3.14 x 64 ≈ 201 square inches.

3. **(a) 67,68,69, . . . 77, 78; b) 186,660)** Students can explore number patterns to determine the first and last numbers in the nth row. The 72nd row begins with 2557 and ends with 2628. Its sum can be found by brute force addition, or by exploring number patterns for arithmetic series.

4. **(641 x 93 = 59,613)** Students will use trial and error to solve this problem. Further exploration of similar problems will reveal a strategy using place value logic to determine the greatest product given n digits.

5. **(24 boys)** Thirty boys play football, and six do not play basketball.

6. **($3.55)** Computing: ($10.45 - $1.25) ÷ 4 + $1.25 = $2.30 + $1.25 = $3.55.

equal share + extra pizza = Johna's share.

7. **(14)** The first seven balls could all be green, and the next five could all be red. Therefore you must draw two more to be sure you have two of every color.

8. **(-8˚ F)** Readings:

1st	2nd	3rd	4th	5th
22˚	22 + 10 = 32˚	32 - 25 = 7˚	7 - 15 = -8˚	-8 + 21 = 13˚

9. **(a) 38; b) 30; c) 278; The operation, #, is commutative i.e. a # b = b # a or a x b + 2 = b x a + 2 since multiplication is commutative in the real number system. The operation, &, is not commutative since a & b ≠ b & a or a + b x 2 ≠ b + a x 2. a & b = b & a, if and only if a = b.)** Students will need to carefully apply the order of operations and examine several cases in determining the solution to this problem.

MathStars

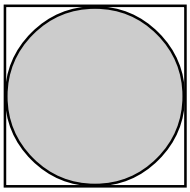
a problem solving newsletter

Vol. 8 No. 3

★★★ 1. The measure of the two acute angles in a right triangle are in the ratio 2:3. What is the measure of the smallest angle of the triangle?

★★ 2. Which two positive integers whose sum is 24, have the smallest product?

★★ 3. If the radius of the circle below is four, what is the area of the unshaded region? (Use $\pi = 3.14$)



★★ 4. If the letters A, B, C, and D represent four different digits, what does A represent?

$$\begin{array}{r} 7\ B\ C \\ +B\ B\ 3 \\ \hline A\ D\ A\ D \end{array}$$

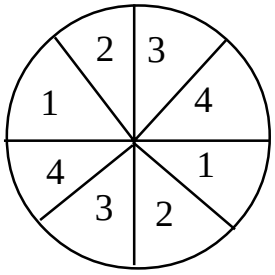
★★★★ 5. Here is a famous sequence of numbers: 7, 10, 16, ___, 52, . . .

a) Find the missing number.

b) Write a rule for this sequence. [Hint: the rule invloves the numbers 1.5 and 4.]

c) This is called Bode's Pattern. Look it up in a science book and explain what it describes.

★★ 6. A game is played with two spinners like the one pictured here:



The sections are equal and the spinners are fair. If each spinner is spun once, what is the probability that the sum of the numbers on the spinners is 6?

Strategy of the Month

Being a problem solver is something like being a detective! A detective has to solve crimes by guessing what happened and checking the guess to see if it fits the situation. For some problems, your best strategy may be to make a guess and then check to see if your answer fits the problem. If not, decide if your guess was too high or too low and then make a second "guesstimate." A good detective keeps records (usually some kind of chart) to help see any patterns and to narrow down the possibilities. You should do this too. The results of incorrect guesses can give you valuable clues to the correct solution. ***Guess and then check*** *the solution to :*

Jo, Ro & Bo have less than 20 tokens. Ro has more than Mo and Mo has more than Bo. Ro gives Bo three tokens and Mo gives Bo two. Now they each have the same number. What did they start with?

MathStars Home Hints

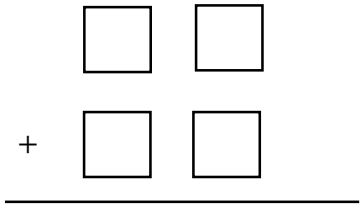
Memorizing number facts will save you time. Flash cards are one way to learn new facts, but you also might try these ideas:

- *play dice or card games in which you need to add, subtract, multiply, or divide.*
- *learn new facts using ones you already know (7+7 =14 so 7+8=15).*
- *learn facts that are related to each other (7x6=42, 6x7=42, 42÷6=7, 42÷7=6).*
- *make a list of the facts you need to memorize and learn 5 new facts each week.*
- *Spend 5-10 minutes every day practicing facts.*

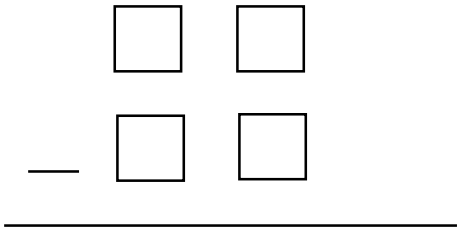
★ 8. Find the next three elements of this sequence:

1, 1, 1, 3, 5, 9, 17, 31, ___, ___, ___, . . .

★★ 7. a) Fill in the squares with four different digits, 0 - 9, to make the largest possible sum.



b) Find four different digits to produce the smallest difference.



Setting Personal Goals

Communicating mathematically means that you are able to share your ideas and understandings with others orally and in writing. Because there is a strong link between language and the way we understand ideas, you should take part in discussions, ask questions when you do not understand, and think about how you would explain to someone else the steps you use in solving problems.

MathStars
Commentary for Teachers
Vol. 8 No. 3

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Discussion of the problems...

1. **(36˚)** The angles are complimentary, therefore their sum is 90˚. A proportion ($\frac{2}{5} = \frac{x}{90}$) or linear equation (2x + 3x = 90) will yield the desired results.

2. **(1 and 23)** Students may wish to explore solution if the conditions are changed. i.e. Which two integers . . . or which two numbers. . ..

3. **(13.76 square units)** The area of the square is 64 square units. The area of the circle is 50.24 square units.

4. **(1)** The largest possible sum for 7 + B has a one in the ten's place.

5. **(a) 28; b) 1.5 x 2^n + 4, where n is the number of the term; c) Bode's Pattern describes the relative distances of the planets from the Sun in our solar system.)** This is a most challenging problem. The actual numbers in Bode's Pattern approximate the relative distances.

6. **(P(6) = 12/64)** There are 64 possible sums, many of which are repeats. The sum six occurs 12 times, therefore, P(6) = 12/64.

7. **(a) 183; 9 and 8 in the ten's place 7 and 6 in the one's place, actual numbers can vary; b) -88; 10 - 98)** This problem can be explored with varying conditions i.e. one or more repeats allowed, only positive integer differences, one decimal point allowed, etc.

8. **(57, 105, 193)** Related to the Fibonacci sequence, this problem affords students some similar explorations.

MathStars

a problem solving newsletter

Vol. 8 No. 4

★★★ 1. The numbers 3, 4, 5 are a fundamental Pythagorean triple because $3^2 + 4^2 = 5^2$. The numbers 6, 8, 10 form a Pythagorean triple but not a fundamental one because they are multiples of 3, 4, 5.

a) List all the fundamental Pythagorean triples with the largest number less than 20.

b) Find a fundamental Pythagorean triple such that the difference between any two numbers is greater than one.

★★ 2. One-fourth is the same part of one-third as one-half is of what number?

★★ 3. Which positive integer less than 100 has the most proper factors?

★★★★ 4. A triangle is a convex polygon that has no diagonals; a convex quadrilateral has two diagonals.

a) List the names of all convex polygons with three to ten sides. How many diagonals does each polgon have?

b) Predict the number of diagonals that can de drawn in a convex polygon with 22 sides.

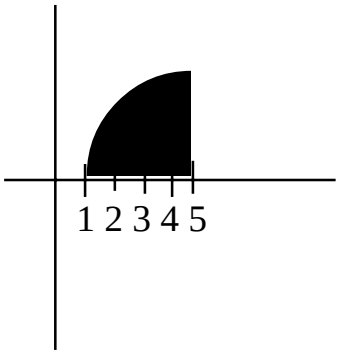
c) Write a rule that gives the number of diagonals for any convex poygon.

★★ 5. Complete the pattern:

$$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \_\_\_, \_\_\_, \_\_\_, \frac{13}{2} \ldots$$

★★★ 6. In the figure below, what is the perimeter of the shaded figure?

(Use $\pi = 3.14$)



Strategy of the Month

Noticing patterns helps people solve problems at home, at work, and especially in math class! Math has been called "the study of patterns," so it makes sense to look for a pattern when you are trying to solve a problem. Recognizing patterns helps you to see how things are organized and to make predictions. If you think you see a pattern, try several examples to see if using the pattern will fit the problem situation. Looking for patterns is helpful to use along with other strategies such as make a list or guess and check. How can ***finding a pattern*** *help you solve this problem?*

If $3^2=9$, $3^3=27$, $3^4=81$, the products have 9,7,1 respectively in the one's place. If you compute 3^{15}, what number will be in one's place? Is there a similar pattern for 4^2, 4^3, 4^4...?

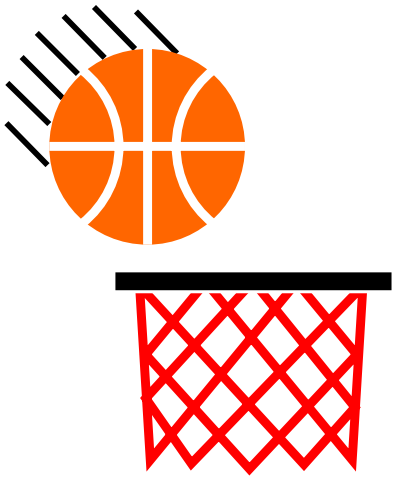
MathStars Home Hints

Set aside a special time each day to study. This should be a time to do homework, to review, or to do extra reading. Be organized and have a special place in which to work.This place needs to have a good light and to be a place where you can concentrate. Some people like to study with quiet music; others like to sit at the kitchen table. You need to find what works for you!

Remember that when you are reviewing or working on solving problems it may help to study in a group.

★★★ 8. Jim and Janie are solving problems in math class using the strategies they have been studying. The current problem they are working on is finding the sum of the first 100 positive even numbers. While Jim is looking for his calculator, Janie notices that there are 50 pairs of sums and uses mental math to find the solution. What did Janie multiply by 50 to find the sum?

★★★ 7. Michael made five baskets and scored 12 points in a recent game. If baskets are worth one, two, or three points, how many different ways could Michael score 12 points?



★★ 9. How many ways can the boxes below be filled with digits, 1 - 9, to make true statements?

$$\frac{6}{\square} = \frac{\square}{4}$$

Setting Personal Goals

If your goal is to become a more <u>responsible</u> student, it means that you

- *actively participate in class.*
- *complete your assignments.*
- *have everything you need in class.*
- *ask for help when you do not understand.*
- *be willing to investigate new ideas.*

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Vol. 8 No. 4

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Discussion of the problems...

1. **(a) (3, 4, 5), (5, 12, 13), (8, 15, 17); b) (8, 15, 17), (12, 35, 37), (48, 55, 73) and many more. . .]**

2. **(2/3)** Since one-half is twice one-fourth, double one-third to arrive at two-thirds, or solve the proportion: 1/4 : 1/3 or 1/2 : x.

3. **(These each have 11 factors: 96, 90, 72, 60)** After examining lists of factors, deficient, abundant, perfect and amicable numbers could be explored.

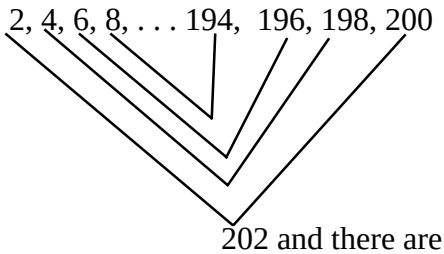
4. **(a) triangle, 0; quadrilateral, 2; pentagon, 5; hexagon, 9; heptagon, 14; octagon, 20; nanogon, 27; decagon, 35 b) 209 c) If n is the number of sides then there will be $\frac{n(n-3)}{2}$ diagonals.)**

5.**(7/2, 9/2, 11/2)** The numerators are the odd numbers while the denominators are all two.

6. **(14.28 units)** The perimeter of the arc is one-fourth the circumference of a circle with radius four; to that length you must add two radii for a total 6.28 + 8 = 14.28.

7. **(There are two ways he could have scored: three 3-pointers, one 2-pointer and one 1-pointer or, two 3-pointers, and three 2-pointers.)**

8. **(202)** She noticed the series 2, 4, 6, 8, . . . 194, 196, 198, 200 and paired the numbers like this:



202 and there are 50 pairs.

9. **(Four ways: 6/3 and 8/4, 6/8 and 3/4, 6/6 and 4/4, 6/4 and 4/6)**

MathStars

a problem solving newsletter

Vol. 8 No. 5

1. ★★ The radius of a tennis ball is x centimeters. If a container holds three balls, express its height in terms of x.

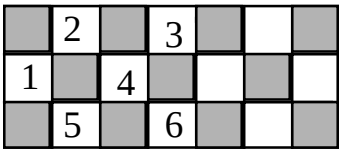
2. ★★ A box is to be built to hold 72 cubic feet of tools. If the rectangular container must be 36" wide and 48" long, what must the height be to accommodate the tools?

3. ★★★ It is possible to write an expression for every number from 1 to 100 using exactly 4 fours and any operations:
[+, -, x, ÷, (), $\sqrt{}$, !]. For example:
$1 = 4 \div 4 \times 4 \div 4$; $2 = 4 \div 4 + 4 \div 4$.
Write an expression for eight using exactly 4 eights and any operations:
[+, -, x, ÷, (), $\sqrt{}$, !].

4. ★★★★ Fill in the blanks with digits from {0,1,2,3,4,5} to make a true expression.

$$\frac{\square \times 3}{2} - \square = 5$$

5. ★ In the figure below, finish numbering the unshaded squares in the pattern as started:



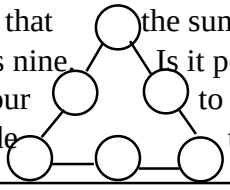
6. ★ A class has 37 students. If I draw names one at a time, how many names must I draw to be certain that

a) at least two of the names drawn have the same birthday month?

b) at least three of the names drawn have the same birthday month?

Strategy of the Month

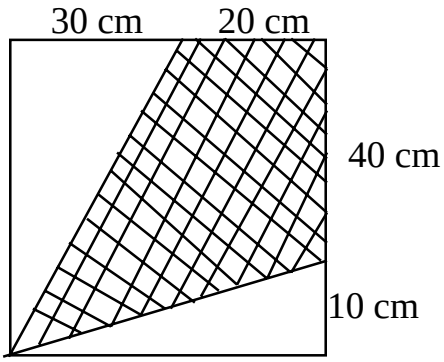
Sometimes mathematical ideas are hard to think about without something to look at or to move around. Drawing a picture or using objects or models helps your brain "see" the details, organize the information, and carry out the action in the problem. Beans, pennies, toothpicks, pebbles, or cubes are good manipulatives to help you model a problem. You can use objects as you guess and check or look for patterns. Try ***using objects*** *to help you solve this problem:* Place the first six counting numbers in the circles so that the sum on each side of the triangle is nine. Is it possible to use the numbers four to nine to make the sum of each side the same?



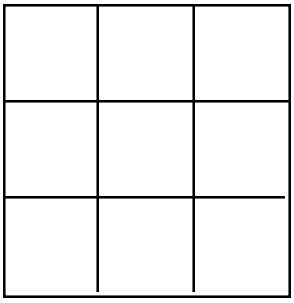
MathStars Home Hints

Remember when you had "Show and Tell" in kindergarten? Now you have a great deal to share in mathematics. Talk to the folks at home about what you are learning. Show them your papers and tell them about what is happening in your math class. Let them see that you are doing problems in class similar to these. Each week choose an assignment that you are proud of and display it somewhere in your house.

9. ★★★ The area of the shaded portion is what percent of the area of the entire square?



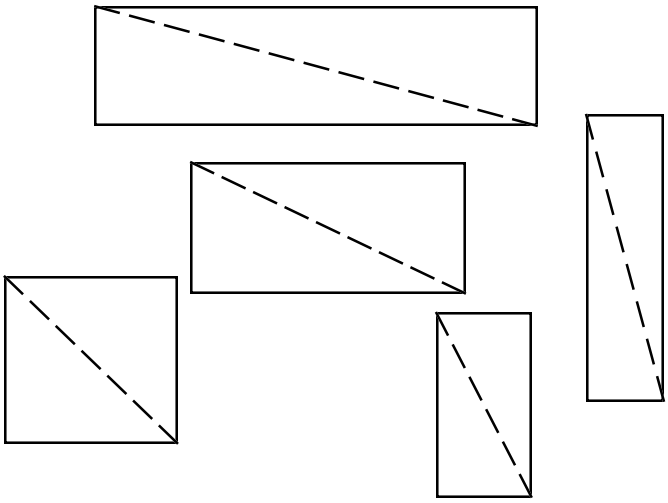
7. ★★★ The figure below is a 3 by 3 square.



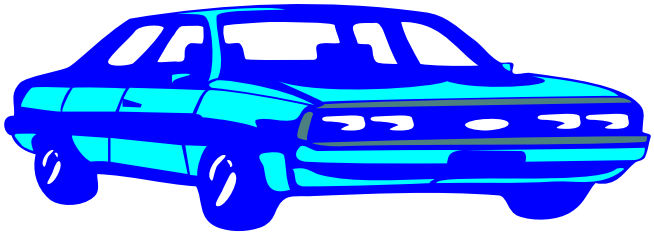
a) How many squares are in the figure?

b) How many squares are in an 8 by 8 square?

8. ★★ The perimeter of a rectangle is 60 centimeters. To the nearest tenth of a cm, what is the length of the shortest possible diagonal the rectangle could have?



10. ★★ In South Carolina, auto license plates consist of three letters followed by three digits. In North Carolina, a license plate has three letters followed by four digits. Any letters or digits can be repeated. How many more plates are possible in North Carolina than South Carolina?



Setting Personal Goals

Mathematics is all around us. We use it every day in personal living and in all of our school work. When we read graphs in social studies, gather and use data in science investigations, or count in music or physical education, we are using mathematics. We make connections in our math classes also; for example, measurement skills help us in solving many geometry problems and classification skills help us in organizing data. We use computation in many different situations. You will become a stonger mathematics student by making connections.

MathStars
Commentary for Teachers
Vol. 8 No. 5

About these newsletters...

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Discussion of the problems...

1. **[6x]** Since the can will hold three balls and the diameter of a ball is 2x, the height of the can must be at least 6x.

2. **[6 feet or 72 inches]** Volume = length x width x height or 72 = 3 x 4 x height. Height = 72÷ 12 = 6.

3. Answers will vary. Here are some : (8 - 8) x 8 + 8 = 8; $8 \times 8^{(8-8)} = 8$; $8 \times (8 \div 8)^8 = 8$; $(8 - 8) \div 8 + 8 = 8$.

4. **[4, 1]** $\frac{(\mathbf{4} \times 3)}{2} - \mathbf{1} = 5$

5.

	2		3		**8**	
1		4		**7**		**9**
	5		6		**10**	

6. **[13, 25]** This is an example of the Pigeonhole Principle. It can easily be modeled with a calendar and a set of student names. a) Once 12 names have been drawn, if each has a unique birthday month, the next name guarantees a match. b) Again, if 24 names have been drawn and they are all paired, then the 25th name will produce a triple.

7. **[a) 14; b) 204]** Students should begin their search with a system or plan. They can start by counting the 1x1's, 2x2's, etc., and look for a pattern. The count will proceed to find one square in a 1x1, five in a 2x2, fourteen in a 3x3, etc., or 1, 1+ 4, 1+ 4 + 9, 1+ 4 + 9 + 16, 1+ 4 + 9 + 16 + 25 + ...

8. **[15√2 or 21.2 cm]** The square will give the shortest diagonal. With perimeter 60 cm, the side will have length 15 cm; the Pythagorean Theorem provides the diagonal length as $\sqrt{15^2 + 15^2}$.

9. **[60%]** The area of one unshaded triangle is .5 x 50 x 30 or 750 square centimeters, while the other triangle has area .5 x 50 x 10 or 250 square centimeters. The square has total area 50 x 50 or 2500 square centimeters. Computing: total area - unshaded area = shaded area or 2500 - 1000 = 1500 or the shaded area is 1500 square centimeters. The percent shaded is therefore 1500 ÷ 2500 or 60%.

10. **[158,184,000 plates]** South Carolina Plates = 26 x 26 x 26 x 10 x 10 x 10 or $26^3 \times 10^3 =$ 17,576,000; while North Carolina Plates = 26 x 26 x 26 x 10 x 10x 10 x 10 or $26^3 \times 10^4 =$ 175,760,000. The difference is 158,184,000.

MathStars

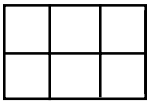
a problem solving newsletter

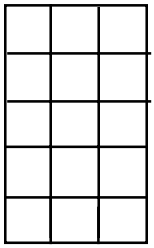
Vol. 8 No. 6

1. ★ A cubic foot is what fraction of a cubic yard?

2. ★★ You have an unlimited supply of 5-cent and 11-cent stamps. Using only these stamps you can pay 16 cents in postage but not 17 cents. What is the largest amount of postage that cannot be paid with any combination of these stamps?

3. ★★★ The figures below are two faces of a rectangular solid. Each small square has an area of one square centimeter.
Find the:
a) volume of the rectangular solid.
b) surface area of the rectangular solid.

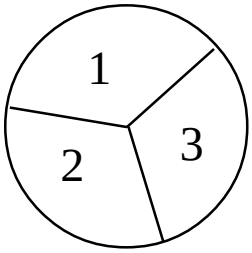




4. ★★★ Two dice are rolled. The sum is not three. What is the probability that one of the dice is a three?

5. ★ What number could you add to 7/12 to get the sum 11/16?

6. ★★★ A game uses the spinner below and a single die to determine its moves. The number of spaces a player moves is the sum of the number on the die and the number on the spinner.



How many ways can a player move five spaces?

Strategy of the Month

When a problem involves data with more than one characteristic, ***making a table, chart, or graph*** *is a very good way to organize the information. It helps your brain to identify patterns and to discover any missing data. Tables help you record data without repeating yourself. Making a table or chart is especially useful for certain problems about probability and for some logic problems. Sometimes tables and charts are included in your information and you need to read through them carefully to understand the data you need to solve your problem. Creating a graph is also a good way to organize and visualize information.* ***Make a table*** *to solve this problem:* A certain factory makes bikes and trikes. Seats come in boxes of six. The foreman hates to have any leftovers at the end of a shift. If 59 wheels are sent by the home office, how many boxes of seats should be ordered?

MathStars Home Hints

Everyone learns from sharing, and you can continue to learn by teaching others about the new mathematics ideas you are learning. Become a teacher and help a younger student. Explain what you have learned and what else you want to know. Good teachers set goals and evaluate the progress made toward reaching these goals. You will continue to be a learner whenever you become a teacher.

7. ★★ The Blue Ridge Mountain Outdoor Club is going rafting. They can rent 5-person, 7-person, and 8-person rafts. All the rafts must be full in order to have the correct balance. The daily rental rates are:

5-person raft	$24.00
7-person raft	$28.00
8-person raft	$34.00

If there are 89 members going on the trip, what combination of rentals will be the least expensive for the club? What will the total cost be for a one-day trip?

8. ★★ What figure is formed by connecting the following points in order:
A (5,1), B (9,1), C (10,3), D (6,3) ?

Square? Trapezoid? Parallelogram? Rhombus? How do you know?

ANSWERS

1. _____
2. _____
3. a)_____
   b)_____
4. _____
5. _____
6. _____
7. _____
   _____
8. _____

Setting Personal Goals

Perseverance means that you do not give up easily. Good problem solvers try different strategies when they are stumped and are not discouraged when they cannot find an answer quickly. They stick to the task, using all of their previous experiences to make connections with what they know and the problem they are trying to solve. If something does not work, they discard the unsuccessful idea and try again using a different strategy.

MathStars
Commentary for Teachers
Vol. 8 No. 6

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Discussion of the problems...

1. **[1/27]** A cubic yard is 3' by 3' by 3' or 27 cubic feet.

2. **[39 cents]**

3. **[a) 30 cm^3; b) 62 cm^2]** The figure measures 2 by 3 by 5 centimeters.

4. **[11/36]** This answer assumes we are considering at least one die is three. If the question considers exactly one die is three the probability is 10/36.

5. **[5/48]** Solving the equation 7/12 + n = 11/16, the value for n = 11/16 - 7/12 = (33 - 28)/48 or 5/48.

6. **[3]**

Spinner	Die
1	4
2	3
3	2

7. **[Twelve 7-person rafts and one 5-person raft is the cheapest combination at $360.00 per day]** Making a chart of configurations and cost is a good strategy for students to organize their work.

8. **[parallelogram]** Reasons will vary.

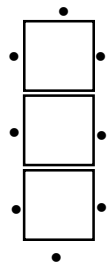
MathStars

a problem solving newsletter

Vol. 8 No. 7

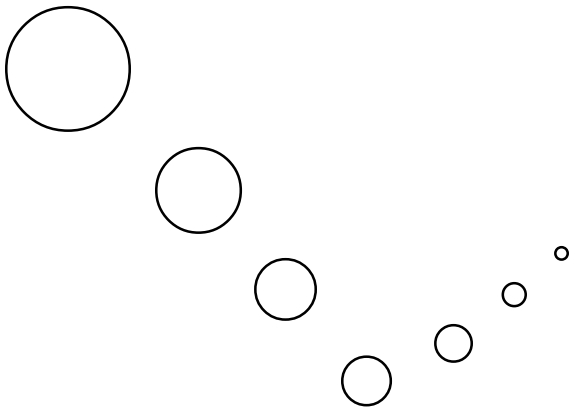
1.★★★ The coach is arranging square tables for the Math Club banquet. Tables pushed together seat one person on a side. Complete the chart to show the number of seats needed.

Tables	Seats
1	4
2	__
3	8
4	__
5	__
20	__
50	__
100	__



Is this reasonable? Why?

2. ★★★ A dropped super ball rebounds one-third the distance of its preceeding height on each bounce. If such a ball is dropped from a window 36 feet off the ground, how high will it rebound after three bounces? after 10 bounces? after 100 bounces?



3.★★ Nathan, Dwayne, John and Allen each have a favorite sport: volleyball, track, swimming and football. Nathan and Allen both use balls in their sport, John is afraid of deep water, and Nathan hates to wear a helmet. What is each boy's favorite sport?

4. ★ A softball team has five pitchers and three catchers. If pitchers never catch and catchers never pitch, how many different pitcher - catcher pairs are possible?

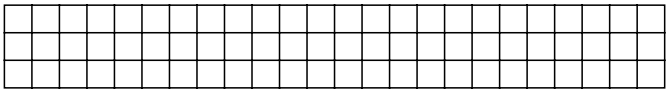
Strategy of the Month

Some problems are difficult to "see" even if you draw a picture. For these problems, it can be helpful to actually ***act out the problem****. When you role play with friends or people at home, you may discover the solution as you act out the problem. Or you may recognize another strategy that will help you find the answer. Sometimes "acting out" a problem can be done with manipulative materials. To find the solution to the problem below, become the director and choose your cast to act this out:* Joey's little sister, Ella, must take four steps for every three steps Joey takes. Suppose one of Joey's steps covers 32 centimeters. How far will Ella travel when she has taken 12 steps?

MathStars Home Hints

Calculators are important tools. They do not replace mathematical thinking; you must tell the calculator what numbers and operations to use. Calculators allow students to focus their energies on solving problems and to easily try alternative solutions. They also allow students to solve problems that were too difficult for pencil and paper. Number sense and good estimation skills are important when students use technology to carry out computations. Explore some "what if" situations with the calculator. "What if the cost of gas goes up 4¢... What if we build the patio 2 feet wider..."

5. ★★ In the figure below, each small square tile has an area of one square unit.



a) Rearrange the small squares, keeping a rectangular shape, but make the perimeter as large as possible.

b) Rearrange the small squares, keeping a rectangular shape, but make the perimeter as small as possible.

6. ★★ If you start at 8 a.m. on Wednesday morning, January 1st and count by ones, one number per second, when (day and time) will you reach one million?

7. ★★★ There are 120 students in the tenth grade class at Mt. Carmel High School. Three science courses are offered; and forty students take Biology, forty-four take Chemistry and thirty-two take Physical Science. Twenty-three students take Biology and Chemistry, twelve take Biology and Physical Science while eighteen students take Chemistry and Physical Science. Three students are taking all three courses.

a) How many students take:

only Biology?
only Chemistry?
only Physical Science?

b) How many students are not taking a science course?

Setting Personal Goals

Accuracy is very important to everyone. Pharmacists must always measure accurately when preparing prescriptions and carpenters must cut supporting boards precisely to fit. Careless mistakes may be avoided in the classroom by computing carefully, checking back over work, and writing numbers clearly and neatly. Remember: If work is worth doing, it is worth doing well.

MathStars
Commentary for Teachers
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Discussion of the problems...

1. **[**

Tables	1	2	3	4	5	20	50	100	
Seats	4	**6**	8	**10**	**12**	**42**	**102**	**202**	**Not very, while the numbers seated is correct, the table would be verrry looong!]**

2. **[1' 4" or 1 1/3 ft.; ten bounces - $36(1/3)^{10}$ ft. ≈.0006 ft.; 100 bounces - $36(1/3)^{100}$ ft.]** This is a good problem to illustrate the value of exponential notation.

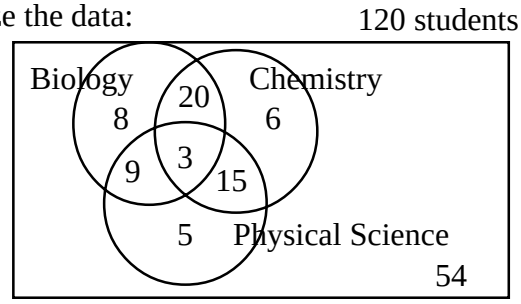
3. **[Nathan - volleyball, Dwayne - swimming, John - track, Allen - football]** This type of problem lends itself to establishing a grid or chart and sorting through the possibilities.

4. **[15 pairs]** Since the players cannot change positions, each pitcher can be paired with 3 different catchers or 5 x 3 pairs.

5. **[a) 146 units - 72 by 1; b) 34 units - 9 by 8]** This is a good problem for exploring factors and organizing data. With a constant area and varying perimeters, students can explore the dimensions and compare results.

6. **[Sunday, January 12th, at 9:46:40 p.m.]** One million seconds is 11 days, 13 hours, 46 minutes and 40 seconds.

7. **[a) Biology - 8, Chemistry - 6, Physical Science - 5; 54 students]** A Venn diagram is the most efficient way to organize the data:

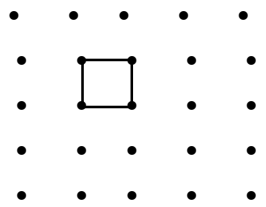


MathStars

a problem solving newsletter

Vol. 8 No. 8

1. ★★ The figure shown on the geoboard below has an area of one square unit. Draw a square with area five square units.



2. ★★★ A unit fraction is a proper fraction with numerator one. Other proper fractions can be written as the sum of unique unit fractions. For example:

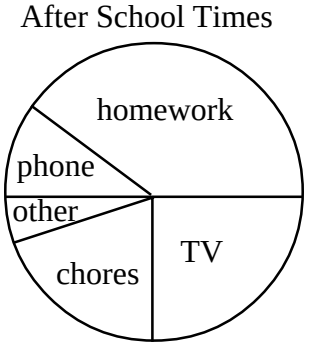
$$\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$$

Write each of the following, in as many ways as possible, as the sum of unique unit fractions:

a) $\frac{5}{8}$ b) $\frac{7}{12}$ c) $\frac{5}{17}$

3. ★ One number is 15 more than another, while their sum is 37. What are the numbers?

4. ★★★ One of Kawana's assignments is to draw a circle graph to represent how she spends her time from 4 p.m. to 9 p.m. on a school night. Here is her graph:



Approximately how long does she spend on homework?

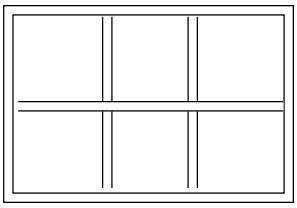
Strategy of the Month

What do you do if you have a problem that seems to be very complicated? It may have a lot of large numbers, too much information, or multiple conditions. One approach is to create a simpler problem like the one you need to solve. As you solve the easier problem, you may see the way to solve the more difficult one. Or you may discover a different process that will work with the harder problem. The trick is to be sure that your simpler problem is enough like the original one that the patterns or process you use will help you with the harder situation. ***Make a simpler problem*** *first as you solve this:* What is the sum of the first 500 counting numbers? i.e., The sum of 1 + 2 + 3 + 4 + . . . + 496 + 497 + 498 + 499 + 500 ? How about only the odds: 1+ 3 + 5 +7 + . . .+ 493 + 495 + 497 + 499 ?

MathStars Home Hints

Math skills develop as you apply concepts learned in school to real life situations. Which product is the best buy? How many tiles will it take to cover the kitchen floor? What time should we start baking the turkey so that we can have dinner at 7 p.m.? What do the statistics tell us about the two baseball players?

5. ★★★ A glass cutter is cutting glass for window panes. He wants to cut the largest possible squares with the least amount of wasted glass. The piece he is working on measures 60 inches by 75 inches. How many equal square panes can he cut from this piece of glass? What will be the dimensions?

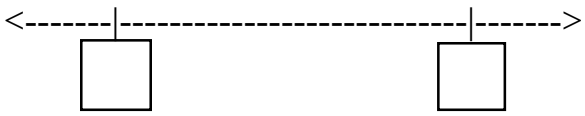


6. ★★★ Simplify each of the following continued fractions:

a) $$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2}}}}$$

b) $$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{5}}}}}$$

7. ★ The $\sqrt{151}$ lies between which two consecutive integers?



8. ★★ On a standard, analog clock face, what is the measure of the smaller angle formed by the minute and hour hands at:

a) 9:15

b) 1:00

c) 1:35

Setting Personal Goals

Confidence means that you believe in yourself. You can become a more confident problem solver by learning to use a variety of strategies. If your first idea does not work, don't give up; just try another way! Working with a buddy also helps. You need to remember that there is usually more than one way to solve a problem and that practice always helps us learn.

MathStars
Commentary for Teachers
Vol. 8 No. 8

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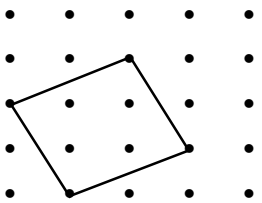
Discussion of the problems...

1. Each side of the square is the hypotenuse of a 1 by 2 right triangle.



2. **[** $\frac{5}{8} = \frac{1}{8} + \frac{1}{2}$ **;** $\frac{7}{12} = \frac{1}{12} + \frac{1}{2} = \frac{1}{3} + \frac{1}{4}$ **;** $\frac{5}{7} = \frac{1}{14} + \frac{1}{7} + \frac{1}{2}$ **]** An interesting follow-up would be a study of Egyptian fractions and some problems from the Rhind Papyrus.

3. **[26, 11]** This problem can be solved by the guess and check method, algebraically : [x + (x + 15) = 37] or graphically : [x - y = 15 ; x + y = 37].

4. **[A little less than two hours]** It appears that homework is three-eighths of the graph which translates to .375 x 5 hours = 1.875 or a lttle over 1 hour and 52 minutes.

5. **[20 square panes, each 15" by 15"]** The greatest common factor, (GCF), of 60 and 75 is 15, i.e., (60, 15) = 15. The area of the glass is 60 x 75 = 4500 square inches. Twenty panes, each with area 225 square inches, use the entire piece of glass, so there is no waste.

6. **[a) 5/8; b) 157/225]** Students may wish to construct their own continued fractions with varying number patterns, compute the values when the fraction has 4, 5, 6 or more stages, and study the sequence of values.

7. **[between 12 and 13]** If students graph the solution, the point should be closer to 12.25 than to 12.5.

8. **[a) 174˚; b) 30˚; c) 162.5˚]** Students can compute the angle by observing that the 360˚ circle is divided by the minute marks into six degree units. They should also observe that the hour hand moves through one of these marks every 12 minutes.

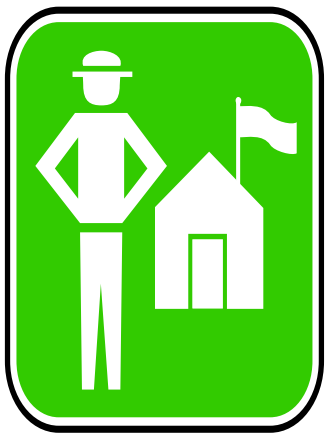
MathStars

a problem solving newsletter

Vol. 8 No. 9

1.★★ A playground director is planning to buy a trampoline. She can choose a rectangular or circular model. The perimeter of the rectangular trampoline is 50 feet and the circumference of the circular model is also about 50 feet. She wants to buy the trampoline with the greater area so the children will have plenty of room to jump and turn. Which model should she buy?

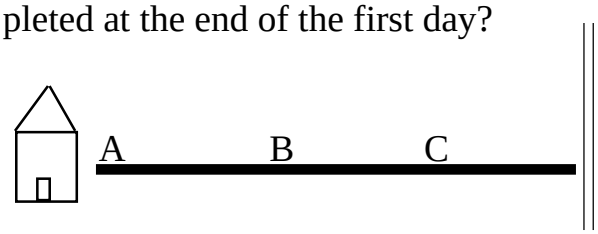
2. ★★★ A group of 53 scouts is planning a camping trip. They can rent 2-person, 3-person, or 4-person tents. Rental costs are equal but they want to take the fewest number possible. If no one wants to sleep in a tent alone, how many of each type should they rent?



3.★★★★ Complete the table below, then write a rule that defines the value of y in terms of x .

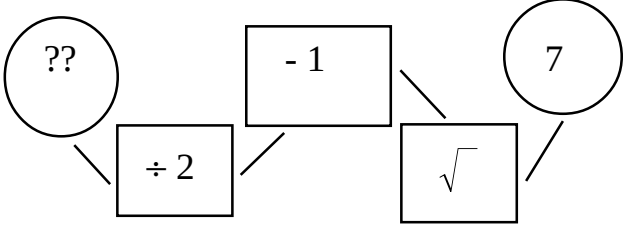
x	1	3	5	7	9	10
y	2	10	26	50		

4. ★★ Doug is digging a ditch from his house to the road. If he digs from A to B the first day, from B to C the second day, and from C to the end on the third day, about what percent of the task will be completed at the end of the first day?



Strategy of the Month

What if you know the result of a situation, but you don't know the beginning? For example, you might know that you end up with thirteen baseball cards after doing a certain number of trades and you want to figure out how many cards you had before the trading started. In that case you need to work backwards; you have to think about your actions in reverse order. This strategy works for any sequence of actions when you know the end result rather than the starting place. Try ***working backwards*** *to find the starting number on this flow chart:*



MathStars Home Hints

Mathematics can make life easier for you when you become a good estimator. Spatial estimation helps you plan how you will rearrange your furniture or how far to jump to cross a puddle of water. Using estimation helps you know whether you have enough money for your purchases before you get to the check-out line. We become good estimators by practicing. Use your number sense and spatial sense to think about what the answers to problems will be before you start to solve them.

5.★★★ Using the digits 1, 2, 3 (without repeats) and any operations write the:

a) largest possible real number

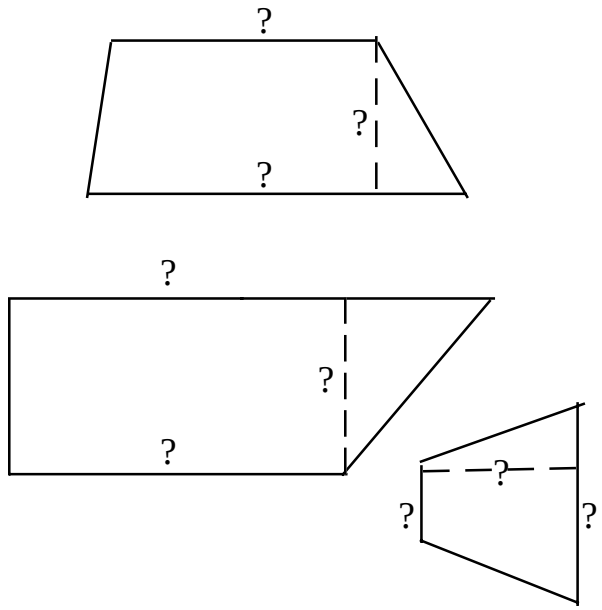
b) smallest positive, real number possible.

6. ★★★★ A certain antibiotic has a half-life of two hours. This means that every two hours, half of the amount of the medicine is available in the bloodstream. At 10:00 a.m. a patient receives 500 mg of the antibiotic.

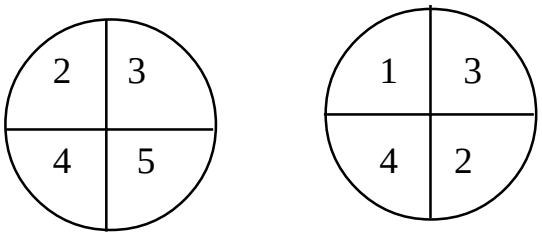
a) How much of the antibiotic remains at 10:00 a.m. the next day if no more serum is administered?

b) The antibiotic is effective only when more than 100 mg is in the bloodstream. If the first dose is given at 10:00 a.m., when should the next dose be given?

7.★★ Find three trapezoids with whole number base and height measurements and with an area of 60 square units.



8.★★ A game is to be played with the two spinners pictured here. List all the possible sums and the probability of each.



Setting Personal Goals

When you encounter a new situation, you use all of your previous experiences to figure out the current problem. Reasoning mathematically means using your brain power to think logically and sequentially, to put prior knowledge with new information. Set the goal of developing mathematical power and use your thinking power to achieve the goal!

MathStars
Commentary for Teachers
Vol. 8 No. 9

About these newsletters...

The purpose of the MathStars Newsletters is to challenge students beyond the classroom setting. Good problems can inspire curiosity about number relationships and geometric properties. It is hoped that in accepting the challenge of mathematical problem solving, students, their parents, and their teachers will be led to explore new mathematical horizons.

As with all good problems, the solutions and strategies suggested are merely a sample of what you and your students may discover. Enjoy!!

Discussion of the problems...

1. **[circular trampoline has greater area, ≈ 199 square feet]** This problem gives students the opportunity to explore equal perimeter figures. A good follow-up would be equal area figures and exploration of varying perimeters.

2. **[14 tents: either twelve 4-person, one 3-person and one 2-person tent, or eleven 4-person and three 3-person tents]** A chart or list will help students organize their work for a guess and check strategy.

3. **[82, 101: $y = x^2 + 1$]**

4. **[about 33%]** Doug dug about one-third of the distance from his house to the road on the first day.

5. **[a) $21^3 = 9261$; b) $21^{-3} = 1/9261 \approx .0001079$]** Students may need to be reminded that zero is neither positive nor negative.

6. **[a) $500 \times (1/2)^{12} \approx .12$ mg; b) shortly after 2 p.m.]** At 2:00, 125 mg of the antibiotic remains in the patient's bloodstream.

7. Answers will vary. Some possibilities are:

h	10	8	6
b_1	7	8	11
b_2	5	7	9

8.

+	1	2	3	4
2	**3**	**4**	**5**	**6**
3	**4**	**5**	**6**	**7**
4	**5**	**6**	**7**	**8**
5	**6**	**7**	**8**	**9**

$P(3) = \frac{\mathbf{1}}{\mathbf{16}} = P(9)$

$P(4) = \frac{\mathbf{2}}{\mathbf{16}} = P(8)$

$P(5) = \frac{\mathbf{3}}{\mathbf{16}} = P(7)$

$P(6) = \frac{\mathbf{4}}{\mathbf{16}}$

MathStars

a problem solving newsletter

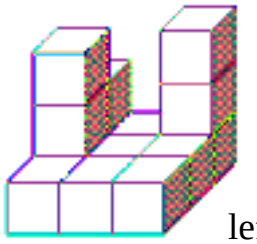
Vol. 8 No. 10

1. ★★ A geometric sequence is a sequence in which the nth term is found by multiplying the term before it by a constant ratio.

a) Find the next three terms in this sequence: 144, 48, 16, . . .

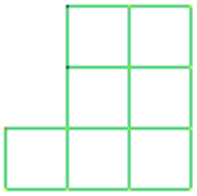
b) Find the 22nd term in this sequence: 2, 5, 12.5, . . .

2. ★★★ Using cubes, build this model:



left

back



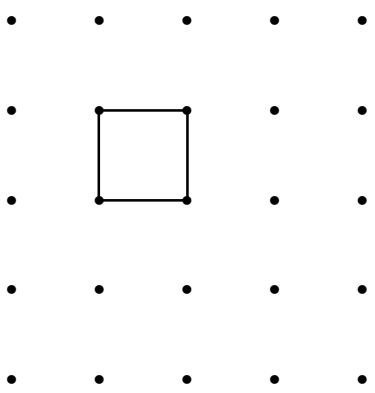
left side view

a) On grid paper, draw the right side view, the back view, and the front view. The left side view is shown above.

b) If each small cube has a volume of one cubic unit, find the surface area of the model.

3. ★ A set of books can be separated into six equal piles, and can also be divided into 15 equal piles. What is the smallest number of books in the set?

4. ★★★★ The square drawn on this geoboard has an area of one square unit.



Which squares, with integer areas of ten or less, cannot be drawn on this geoboard?

Strategy of the Month

You have tried many ways to solve problems this year. Already you know that when one strategy does not lead you to a solution, you back up and try something else. Sometimes you can find a smaller problem inside the larger one that must be solved first. Sometimes you need to think about the information that is missing rather than what is there. Sometimes you need to read the problem again and look for a different point of view. Sometimes you need to tell your brain to try to think about the problem in an entirely different way - perhaps a way you have never used before. Looking for different ways to solves problems is like brain-storming. Try to solve this problem. You may need to ***change your point of view***. Bob and his dad visited the hardware store on Saturday. They observed the following transactions: Mr. Harris bought 4 for $.50, Mrs. Clarke bought 21 for $1.00 , Mr. Montaro bought 1134 for $2.00 and Ms. Park bought 450 for $1.50. What were they buying?

MathStars Home Hints

Identifying the mathematics that is all around you can be lots of fun. Think about the geometry and spatial visualization you use in playing video games or when you play golf or basketball. When your parents parallel park, they are using their spatial skills too. When you track a hurricane, you use coordinates. When you check the stock market or read the latest sports statistics, you are using mathematics. With your family or friends go on a math scavenger hunt. Who can identify mathematics in the most unusual places?

5. ★★ Fill in the blanks using digits from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, with no repeats, to make the smallest possible sum.

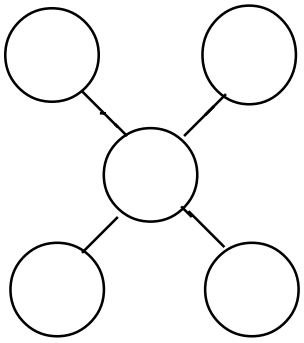
$$\begin{array}{r} .\square\square\square \\ +\ .\square\square\square \\ \hline \end{array}$$

6. ★★ Heather is figuring her income tax for the year. She earned $5,367. The table said that for incomes between $5000 and $6000, the tax is $200 plus 15% of any amount over $5000. How much tax will Heather owe?

7. ★★★★ Complete the table below, then write a rule for finding y in terms of x.

x	1	2	3	4	5	6
y	2	8	18	32		

8. ★★ Arrange the digits {1, 2, 3, 4, 5} in the diagram below so that the sums along each of the segments are equal.



4

3

5

1

2

Setting Personal Goals

Students who recognize the value of mathematics are well on their way to becoming mathematically powerful citizens. Valuing mathematics means that we appreciate the richness, power, and usefulness of mathematics. Without math there would be no roads or bridges, computers or movies, banks or fast food restaurants. How can you become mathematically powerful?

MathStars
Commentary for Teachers
Vol. 8 No. 10

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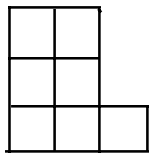
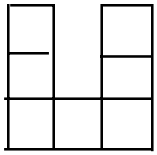
Discussion of the problems...

1. **[a) 16/3, 16/9, 16/27 ; b) 2 $(2.5)^{21} \approx$ 454,747,350.9]**

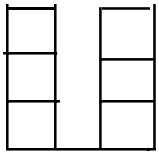
2. **[a)**



right view



back view



front view

b) 47 square units]

3. [**30 books]** An example of an application of least common multiple (LCM), i.e., [6, 15] = 30.

4. **[squares of area three, six, and seven cannot be drawn]**

5. **[.159]** The addends will vary; in tenths place the digits 0 and 1, in hundredths place the digits 2 and 3, and in thousandths place 4 and 5.

6. **[she owes $255.05]** Compute $200 + (.15)($367).

7. **[50, 72; $y = 2x^2$]**

8. **[answers will vary; 5 in the center circle, with 4 and 1 in one line and 2 and 3 in the other]** The common sum is 10.