Grade Four

Classroom

Strategies

1

The learner will read, write, model, and compute with non-negative rational numbers.

Notes and textbook references

1.01 Develop number sense for rational numbers 0.01 through 99,999.

a.) Connect, model, number word, and number using a variety of representations.

A. Make "checkbooks" for each student. Give each student a beginning balance and allow them to earn additional deposits for doing jobs in the classroom, bringing articles or books from home to contribute to class discussions, interesting or especially creative writing, etc. Students would write checks to "rent" their desks, use pencil sharpeners or the water fountain, "purchase" construction paper and "lease" manipulatives. See Blackline Masters I - 2 and I - 3.

You could create a complex classroom economy and carry the project through the semester.

B. Whenever important dates in North Carolina history are discussed, have the students write them in both standard and word form. Talk with children about the way we note years as B.C. and A.D. and what 1992 means (i.e. one thousand nine hundred ninety-two years anno domini *in the year of the Lord*).

Notes and textbook references

***PICO** is a prefix meaning one trillionth or 10 to the power - 12.*

***FERMI**, Enrico, was a famous nuclear physicist. Fermium is a radioactive metallic element.*

***BAGEL** is a hard roll with a center hole, much like a zero.*

C. Play **Bagels**, see p. 64 of Family Math, Lawrence Hall of Science, University of California, Berkeley. This game has been around a long time and many of the early public domain software programs include a version of this game. It has also been called "Fiddle Faddle Flop", see p. 82 of Math for Girls and Other Problem Solvers, Lawrence Hall of Science, University of California, Berkeley. A "leader" chooses a secret three-digit (or four-digit, or five-digit, etc.) number with all the digits being different. The players guess numbers and the leader supplies clues. The goal is to guess the leader's secret number with as few clues as possible. The clue responses include the following: BAGELS means none of the digits in the guess is correct, PICO means one of the digits is correct but is in the wrong place, and FERMI means a digit is correct in the correct place. For example, if the secret number is 427 another player guesses 109, the leader responds with "BAGELS" indicating that none of the digits is correct. If the player guesses 123, the leader responds with "FERMI", indicating that one digit is correct and in the correct place. If the player guesses 145, the leader responds with "PICO" indicating that one digit is correct but in the wrong place. If the player guesses 265, the leader responds with "PICO", indicating that one of the digits is correct but not in the right place. If a player guesses 426, the leader should respond "FERMI FERMI", indicating that two of the digits are correct and in the correct place. Be sure to discuss strategies for making "good guesses" in order to maximize the information gained and minimize the number of clues needed.

Guess	**Response**	
109	Bagels	thus eliminating 1, 0, & 9 from all places
123	Fermi	one digit is correct & in the right place ... it must be 2 or 3 since the first guess eliminated 1
145	Pico	one digit is correct but in the wrong place ... must be 4 or 5
265	Pico	one digit is correct but in the wrong place ... could be 2, 6, or 5
353	Bagels	3 & 5 are now eliminated along with 6 ... thus, 4 & 2 must be in the number ... the 2 must be in the tens place and the 4 is not in the tens place
426	Fermi Fermi	two numbers are correct and in the correct spot ... must be 4 and 2, thus the number is 42_ and this last digit can't be 0, 1, 3, 5, 6, or 9 ... this last digit must be 7 or 8

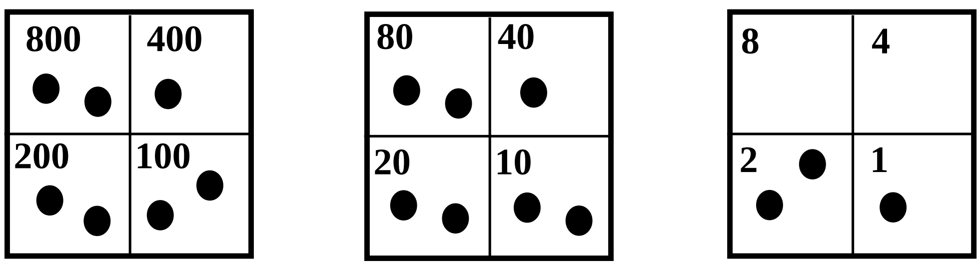
Notes and textbook references

427 Fermi Fermi Fermi Aha, the secret number!

While following logical thinking process associated with this example, go back to all previous clues in order to verify the conclusions.

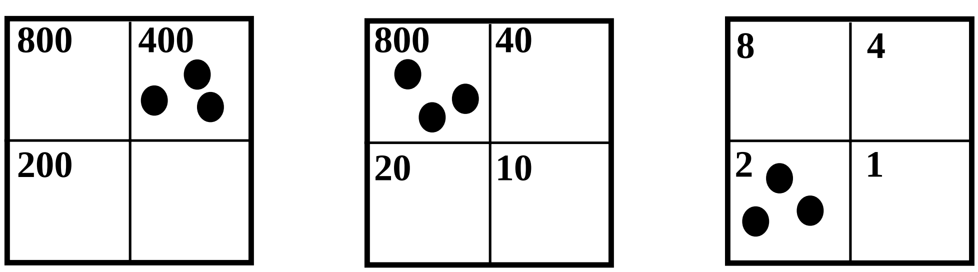
Try playing Bagels with more digits!!

D. (See Blackline Masters I - 21 through I - 24) to learn how to use **Papy's Computer**, sometimes called the minicomputer, to model numbers and compute in non-traditional ways which develop mental math skills in students. Use the boards to represent a "number of the day" each day or weekly for the children to figure out. For example, this configuration of chips shows the number 2,795.



A fourth board showing 1000, 2000, 4000, and 8000 can be added to increase the size of possible numbers.

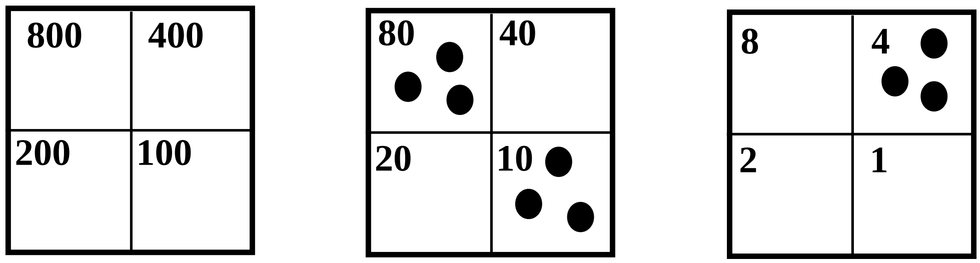
Multiplication can also be modeled on the minicomputer boards by placing more than one chip on a square. The multiplication problem 3 x 482 is shown this way: Chips can be traded as described in the Blacklines so that you end up with only one chip in a square to show the standard configuration of the product.



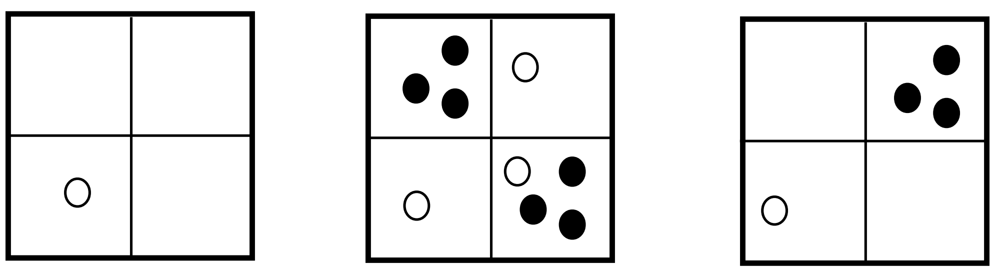
Notes and textbook references

Add:

Papy’s computer may also be used to model multiplication.
For example 3 x 94 would be shown as:



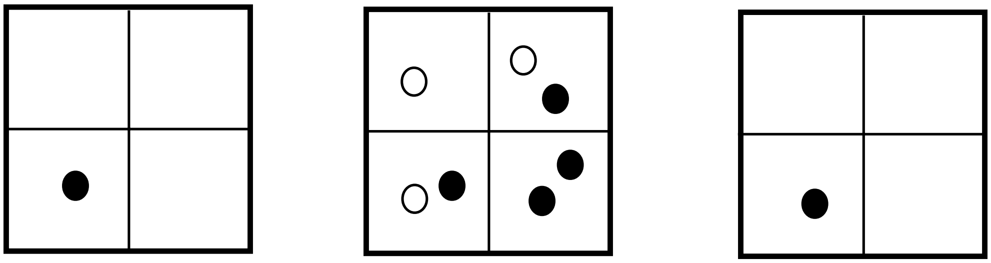
Then begin to trade chips to get to a standard configuration:



(Moved chips are unshaded circles.)

3 x 80 = 240, so one chip on 200 and one on 40
3 x 10 = 30, so two chips on 10 become one on 20
3 x 4 = 12 so one chip goes to 10 and one chip goes to 2.

Now further moves are made:



The two tens plus the one twenty become forty, and the two forties become eighty, so the product is 282.

Many children can learn to mentally make the moves and find the products. This kind of activity helps to develop number sense and is an interesting challenge for many students.

E. **Broken Calculator:** Students work to make a number appear on the display of a calculator without using certain keys, which are said to be “broken”. Broken keys might be digits, operation buttons, or both. Encourage multiple solutions.

Pose problems. Make 1000 but the 1 and 0 buttons are broken.
Students solve the problem; Share or record solution.
Teacher might record solutions to enable students to see patterns that emerge.

Possible problems:
Try to make 88 but the 8 key is broken.
Try to make 0.25 but the 2 and 5 are broken.
Try to make 0.25 and + is the only operation button that works.
Try to make 0,25 and ˜ is the only button that works.
Try to make 1000 but the 1 and 0 keys are broken.
Try to make 0.5 without using the division key ÷ or /.

Notes and textbook references

Notes and textbook references

b.) Build understanding of place value (hundredths through ten thousands).

A. Each student needs digit cards: 0 to 9 (Blackline Master I - 1). The same activity can be completed many different times and at different levels of difficulty by drawing 3, 4, or 5 digit cards. Have students place their cards face down and draw a certain number. Extra cards remain in a pile to the side. Using only those cards drawn, ask students to:

- create the largest possible number
- create the smallest possible number
- write the number words for the number created above
- tell what 10 more than their number would be; 100 more; 10 or 100 less
- compare number with partners (determining whose is larger, smaller, middle number)
- build number closest to 500 (5,000 or 50,000)
- make an odd number
- make a number that is a multiple of 5
- create a number between 100 and 400 (1,000 and 4,000 or 10,000 and 40,000)

B. On a rainy day, play “What’s My Number?” Each student writes a 4 -digit number on a card. These are pinned or taped to other students’ backs so that children cannot see their own numbers. Students move around the room asking questions (which may be answered by “yes” or “no”) to guess their number.

C. Have students cut out pictures from magazines which show groups and identify fractions that describe the pictures. For example, you have a picture of four girls standing in one group. One girl has pigtails, but three-fourths of the girls do not have pigtails.

D. Using 3-inch by 5-inch index cards with the 3-inch side on top and bottom, each student writes 10 each of the numerals 0 through 9. Thus, each student has ten 0's, ten 1's, ten 2's, etc. through ten 9's. Then each student tapes the following pieces of colored construction paper into a long strip: a 5 inch by 9 inch blue piece. When these three strips are taped into a strip, it is 5 inches high and 27 inches long with the colors blue, red, and green sequenced from left to right. (Note: any three colors can be used as long as every student uses the same three and in the same sequence.) Next, each student draws vertical lines to separate this strip into 3-inch wide sections. Each color now has three sections that measure 3 inches by 5 inches, matching the size of the index card numerals. Students write a decimal point on the line separating the red section from the green section. This creates a "playing mat" with a thousands period which is blue, a ones period which is red, and tenths and hundredths to the right of the decimal which are green. Students should write their names on these materials and they might be laminated for extended life. (Note: have some students make extra materials for loaning to those who "forget or misplace" theirs.)

Introduce **Big Four**, the goal being to create the largest possible four-digit number. Each student removes one each of the digits 0 through 9 from his/her deck of cards (Blackline Master I - 1). The cards are shuffled and turned upside down in a stack near the "playing mat". Give the signal "flip" (or some other term) and each student turns the top card over and places it on the mat in one of the places of a four-digit whole number. Once the card is placed, it cannot be moved. After 4 cards are flipped and placed, students compare numbers to see who has created the largest number. Students could be asked to write their number in a math journal in both standard and expanded form and using the number words. They might also be asked to write everything they can about a multiple of 3, 5, etc. Students might describe which two numbers they would switch in order to create a larger number. Be sure to lead a discussion about decision strategies for placing numerals on the playing mat.

This game can be varied by flipping four or five cards to create up to a five-digit number. Different sets of numeral cards might be pulled and shuffled at the beginning. The game might be played in rounds. Students' scores for a round are determined by finding the difference between their own number and the largest one created. For example, if the largest number created is 9,645 and a particular student created 9,543, this particular student's score for that round would be 9,645 - 9,543 = 102. After 5 rounds, the student with the lowest total score is "the winner". Another variation would be to change the goal to creating the smallest number or the number closest to a preset target like 500. Students might also be allowed to flip 6 numerals to create a five-digit number with the option to discard one of the numerals flipped.

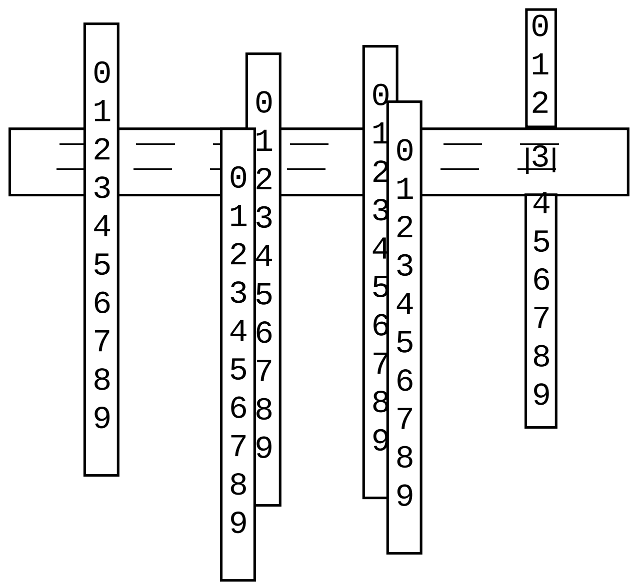
Notes and textbook references

Notes and textbook references

E. What is the population of the state of North Carolina? Which digit is in the ten thousands place? the hundreds place?

F. How much is a thousand? Use base 10 materials and cubes made from centimeter grid paper (Blackline Master I - 25) to model a thousand. Using these cubes, determine how many flats (models of 100) it takes to make ten thousand; how many longs (models of 10) it takes to make ten thousand. How many of these cubes would it take to model a million? What process did you use to solve this problem?

G. Have each student make an odometer which could go up to 100,000 miles. Use large index cards or pieces of old file folders and paper strips to slip through as the numbers. Ask students to explain why each strip needs only 0 through 9 on it.



H. How many different ways can you model four-digit numbers using base 10 materials? Make the number 1,358 using base 10 blocks. For example:

one cube three flats five longs eight ones	or	thirteen flats four longs eighteen ones

Notes and textbook references

I. Explore place value with a calculator. Have students display a number such as 247 on their calculators. Ask them to tell their partners what they could do to change the 4 to a 5. They are not allowed to clear the display and re-enter the number. Or your direction might be for them to find a way to show a 0 in the hundreds place. Be sure to have students tell their plan before they enter anything so that both partners can see whether the plan works. Repeat the process many times with three-digit numbers before going to four- and five-digit numbers.

J. Have five to seven students stand at the front of the class. Give each student a large card with one digit (hand out numbers randomly). The leader asks the class specific questions such as, "Which student is in the thousands place? What place value is John? Who can read the number we have made? How could we rearrange the students to show the smallest possible number? the number closest to 3500?" "What if you had that number in dollars, how many ten dollar bills would that be?"

K. Using number tiles 0 to 9 (Blackline Master I - 1) or for a concrete level, use place value mats and base 10 materials and have children show the numbers you call out. For example: Show me a number that is less than 14,573. Show me a number that is greater than 51,342 but less than 61,436. Show me an odd number that is greater than 81,426 but less than 91,458. Show me a number that is one less than 62,789. Notice that some "show me" statements have one correct answer, while others are more open-ended!

L. Use paper models for units and tenths on the overhead (see Blackline Master I - 4 through I - 6) and allow students to model and record numbers such as 3.2, 4.7, and 1.8. When students are comfortable with the model, tell them you are going to cut one of the tenth models into ten parts. How would you write this? (0.01) What would you have if you cut all 10 tenths into 10 pieces each? (100 hundredths). Use your unit, your tenths, and the hundredths pieces to model numbers like 1.14 and 2.07. Recording what you have modeled in three ways will help students relate the new decimal recording system with more familiar words and fraction notations.

M. Have students write about decimals to give you information about their understandings and to give them practice expressing mathematical ideas in narrative form. Answer a single question frequently rather than have long writing assignments. "Explain what a decimal number is." "Why would you prefer to have 2 candy bars rather than 0.8 candy bars? What does 0.4 mean?"

Notes and textbook references

No matter how good our explanations, many students are surprised to find that 0.57 + 0.43 = 1. Somehow, the activities fall short of helping some students group the extension of our place value system to tenths, hundredths, and thousandths. They want to say 2 is less than 0.13 "because 13 is bigger than 2."

N. Use the base 10 blocks to model numbers such as 251 or 372. Suggest that you would like to try substituting a cube of cheese for a one centimeter block used to model the ones (units). Model other numbers such as 586 or 144 with the cheese. Tell students that you are planning to cut one of the little cubes into 10 parts. (Do this on a sheet of clear plastic on the overhead so everyone can see, or in small groups. Try to get the 10 pieces as equal as possible.) Do you still have 144? (Yes) Give 4 students a tiny piece. Now you have 143.6. Show students that this number can be represented in a variety of ways:

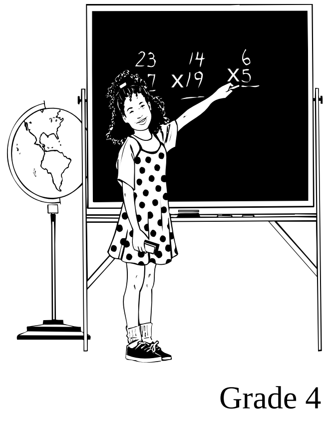
- with words (one hundred forty-three and six tenths)
- as a decimal number - 143.6
- as a mixed number ($143\frac{6}{10}$)
- with blocks

Use the base 10 blocks and the tiny tenth pieces to model other numbers, having students write on their papers (while one student writes on the board) the names of these numbers in words, as mixed numbers, and as decimal numbers.

Give each group of students wax paper to work on, a table knife (plastic), and enough cubes of cheese for each group to have 4. Have students cut one cube into ten pieces. The fact that they are small may help students remember that tenths are parts of one. Ask students to take turns being the leader and naming numbers to model (2.3, 1.7, 3.5, etc.). Students should model the numbers and write them in words, as mixed numbers, and in decimal form.

When you are ready to end the lesson using the cubes of cheese, give the entire group these directions to "dispose" of the manipulatives. Keep in mind the primary purpose is to model decimal numbers and relate the models to the manner in which they are recorded.

- Show me 1.2 cubes of cheese. Remove these. What do you have left? (2.8)
- Show me 1 cube. In a different place show me 0.4 of a cube. Which is more (greater), 1 or 0.4? Remove 0.4. What is left? (2.4)
- Remove 2. What is left? (0.4) Write on your paper the symbols which tell how many cubes are left. (You may wish to show students that some people would write .4 while others would write 0.4. Ask students to explain why both are correct.)



Notes and textbook references

O. Give each student a paper model (see Blackline Master I - 4 through I - 6) for 2 units, tenths, and hundredths and a sandwich bag to store the models. Have students cut apart the tenths and the hundredths. Have them practice modeling numbers and recording them in three ways. You might have students work at their desks and different ones take turns working at the overhead and writing on the board. Toward the end of the lesson, give students four decimal numbers for them to make with their models; draw a picture of the models on their paper, and record in three ways (in words, as a mixed number or fraction, and in decimal form).

P. Have students make many other models for tenths. For example, in South America egg cartons come with places for 10 eggs. Students could cut special models. Students could make additional fraction bars for tenths (they may need help in dividing the bars into 10 parts). Have students look for things which come in tenths and create a bulletin board showing the models and three ways to record each.

Q. Allow students to work in groups to create a decimal model set (see Blackline Master I - 4 through I - 6) . You may wish to duplicate the decimal models on card stock. A complete set will have 33 cards. There will be one unit card, 11 cards colored as tenths, and 21 cards colored as hundredths. Each group may wish to identify their set by putting a design or their initials on the back of their cards. Students should use light colored crayons, markers, or coloring pencils so that the divisions will show. Color the following:

0 tenths (0.0)	a unit (1)
2 tenths (0.2)	1 tenth (0.1)
10 tenths (1.0)	0 hundredths (0.00)
5 hundredths (0.05)	10 hundredths (0.10)
15 hundredths (0.15)	90 hundredths (0.90)
95 hundredths (0.95)	100 hundredths (1.00)

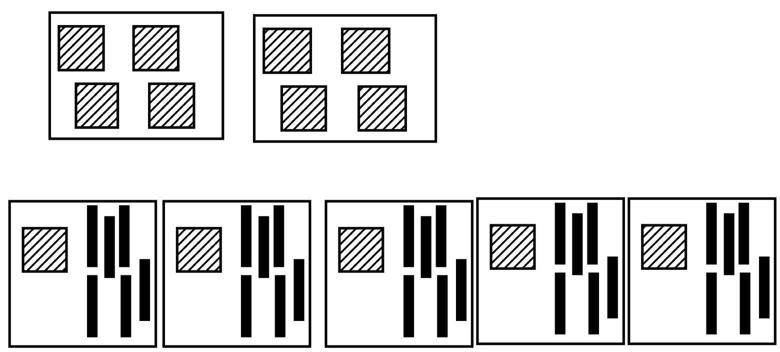
R. Students, working in groups, use one set of decimal models (See Blackline Masters I - 13 through I - 17) to play an inequality activity called "**War**". The models are equally dealt out to the players (extras go in a discard pile). Each player's stack is face down. Students simultaneously turn over the model on the top of their stack. Comparing the face-up models, the player that has a greater amount shaded takes the other models. If there is a tie on the shaded amount then those students spell the letters in **War** as they turn over a model for each letter. Whoever has the greatest shaded amount wins all the models face-up. Play continues until one player has all the models.

Notes and textbook references

S. Extend the “**War**” card game by adding cards with the decimal numbers which match the models written on them. Shuffle these with one deck of picture cards. Follow the same rules for playing.

T. Use decimal models and calculators to discuss what it means when the remainder in a division problem is shown as a decimal instead of a whole number. For example, model and use the calculator to solve 8 ÷ 2 = 4. The 8 squares are easily shared equally in two sets. Now show 8 ÷ 5 = 1 remainder 3 or 1.6. In this latter example, one square goes in each set but the remaining three squares are cut into 10 tenths for a total of 30 tenths. These 30 tenths are shared equally in the five sets. The calculator does this (divides the remainders into tenths and shares them equally) automatically. If you have access to fraction calculators, students can see the remainders displayed as whole numbers or the quotient in decimal form.

Ask the librarian to talk with the class about how the Dewey Decimal System works.



U. Play “**Beat the Calculator**.” Make pairs of cards showing expanded forms of numerals (some easy, such as 90 + 9 or 70 + 6, and some more difficult such as 6,000 + 40). Pair students and one will be the “Calculator Kid.” This student will try to show the number on the calculator before another student can write it on the board or overhead. The student who can show the number in standard form first gets the point.

5000 + 3 + 4

20,000 + 60

30,000 + 900

8,000 + 200 + 3

9000 + 50 +7

50,000 + 80

1000 + 200 + 6

V. Record ten large numbers on an audio tape, allowing time for students to listen, write and then listen to check. After individuals have used the tape, give them an answer key to check themselves.

Notes and textbook references

W. **Number Stumpers**: Give students number stumpers to solve. Example

- I am an odd number between 500 and 600.
- I am less than 60 tens.
- I am a multiple of 5.
- The sum of my digits is 13.

What number am I?

Provide 300 charts (Blackline Master I - 41) and begin with smaller numbers to support students having difficulty.

After many experiences, challenge students to write their own number stumpers for others to solve. Students should give 4 to 6 clues. Students should solve their problem to prove that it works.

X. Record ten large digit numbers on an audiotape, allowing time for students to listen, write and then listen to check. After individual have used the tape, give them an answer key to check themselves.

Y. Encourage each student to find multiple ways to represent a number and then post different ways the class found together. Encourage students to look for patterns.

<u>Example</u>: If you decrease the flats by 1 then you increase the longs by 10.

Students can represent ways by drawing representations of cubes, flats, longs, and units.

Notes and textbook references

Z. **Place the Point!** Challenge students to exercise their number sense with statement such as these:

Half of 7 is 35

25 is the same as one fourth

My height is about 182 meters

145 is a little less than one and one half

75 is the same as three fourths

Jim brought a 15 liter soda

Half of 5 is 250

2005 is a little more than $20

Have students write their own missing decimal point statements and exchange with each other!

c.) Compare and order rational numbers.

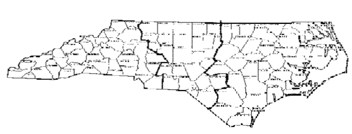
Notes and textbook references

A. Have students find the prices of four similar products (8 oz. bags of cookies, 12 oz. pizzas, new tennis shoes, t-shirts, etc.). Write these on the board in random order and have students rewrite them in order from most expensive to least expensive.

B. Select important dates from North Carolina history. Have students order the dates from earliest to most recent and go on a scavenger hunt to find why the dates are considered special. Discuss the different ways to read numbers; for example, 1952 is read "nineteen fifty-two" as a date but "one thousand, nine hundred fifty-two" in other situations.

C. When students color a North Carolina map to show which counties lie within the 3 (or 4 depending upon what text is used) main regions of North Carolina, have them count the number of counties in each region and write the appropriate decimal. For example: 41 out of 100 counties are in the Coastal Plain region. Another way to say this is 0.41 of the counties in North Carolina are in the Coastal Plain. A more concrete version of this activity would have students begin with a hundreds board and three different colors of cubes. Groups determine a color of cube for each region. Every county within a specific region will be represented by the same color. Place cubes on the hundreds board to represent the counties. For each region count the cubes of each color to tell how many parts of the entire hundreds board was covered by that color. Write the decimal number. Students could then color the map and write the appropriate decimal numbers for each region.

Since North Carolina has 100 counties, using the counties is an ideal way to model decimal numbers less than one. Always relate the examples to parts of the total 100 counties.



D. Students, working together, find out what letters of the alphabet are most frequently used as the beginning letters of the 100 counties' names in North Carolina. Have them decide as a group how they will record their data as they tabulate the number of counties for each letter. Encourage each group to create some type of display and to write the decimals (and fractions) to represent the frequency of letters used in the counties' names.

Notes and textbook references

While students work, you will be able to move around the room and informally assess individuals' understandings of decimal numbers.

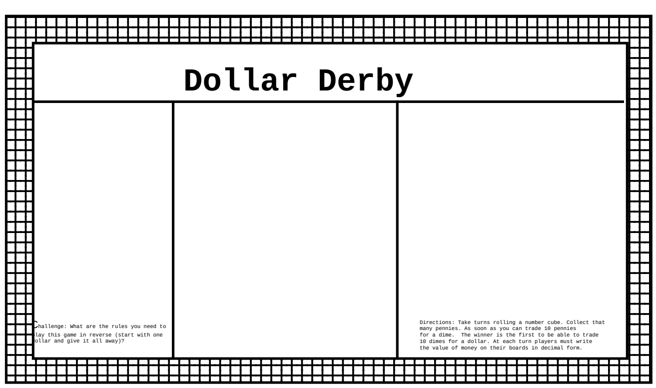
E. Use a population chart from your social studies book or consult the Almanac. Have the students put in order, by population or area, the ten largest cities or counties. Have them identify and order the ten smallest counties.

F. Give each student a set of 12 to 25 index cards and a marker. Ask them to write names of towns in North Carolina whose populations are greater than 1,000 and less than 100,000 on each card (include town and population). When the cards are made, students in pairs or small groups can play "Population Challenge." Shuffle the cards together and then deal out all cards. Each student puts his/her top card down on the table and student with the highest (or lowest) population wins the cards. The winning student must read aloud correctly all numbers or he/she does not get the cards.

G. Reinforce the idea of parts of a whole throughout the year by asking different questions about North Carolina counties and expressing the answers as both decimal numbers and fractions with 100 as a denominator. For example, you might have students do research and determine what part of North Carolina's counties .....

...have populations of less than 50,000.
...border the Atlantic Ocean (or border Virginia...).
...have a county seat (all of them or 100/100 or 1.00).
...have state parks.

H. Play **Dollar Derby**. The object of the game is to be the first to win a dollar. For each group of 3 to 4 students you need a gameboard for each student (see Blackline Master I - 7), one regular die, and play money (pennies, dimes, and one dollar). Notice that the game could be modified to become a subtraction with regrouping game.



I. Use pennies, dimes, and dollars to illustrate decimal numbers. Since students are familiar with 10 dimes to make 1 dollar, dimes are a good model for 0.3 or 0.6 etc. They also know that 100 pennies make 1 dollar. Model that 0.15 hundredths might be shown as 15 pennies (i.e. parts out of the 100 parts - pennies - needed to make 1 dollar) or 1 dime and 5 pennies.

J. Return to the materials developed for 1.01 b) - D. When creating the playing mat, students created a "hundredths period" to the right of the decimal, the green portion of the mat. Actually, this allows for decimal numerals with thousandths. The small 3 inch by 5 inch section on the extreme right could be cut off if this is confusing. Or, this section could be left to encourage discussions. Remember that students have selected certain numeral cards from their decks, shuffled them, and placed this pile of cards beside the playing mat. They flip the top card over and decide where on the mat to place each one. Play a variation of **Big Four** (1.01 b) - D) by having students place the flipped numerals on the ten's, one's, tenth's and hundredth's places to create the largest possible number. Vary this by playing **Little Four**. Extend this activity by using the places on the mat from the hundredths through the hundred thousands to create the largest possible number.

Play a similar game with these same materials in which the goal is to create a number as close as possible to 10, using only 3 flips. After playing the game, have students build their numbers with base ten blocks. In this situation, the small cubes represent hundredths, the longs represent tenths, and the flats represent ones, and the large cubes represent tens. Students can record these models by drawing pictures in their math journals.

K. Play **Higher-Lower**: Within a group one student is the leader and decides on the number limit for the game: 0 - 100 or 0 - 1,000 or 0 - 10,000 or 0 - 1,000,000. The leader writes a number within that range on a piece of paper, folds it, and places it out of sight. Group members take turns guessing the number. The leader repeats the number guessed and tells the group to go higher or lower. The player who guesses the correct number wins and becomes the new leader.

Notes and textbook references

For the math journal...
How do you know that 0.6 is greater than 0.06?

Notes and textbook references

Imagine a student who created the number 5,362. This student enters 5,362 into the calculator and needs to find a way to change the display to 6,432 without clearing the display and entering new numbers. This student might add 1,000, then add 100, and subtract 30. Or, this student might add 1,100 and subtract 30. Or, this student might add 1,070. All of these approaches demonstrate various levels of understanding.

L. Have students use their numeral cards and playing mats for **Who Has the Largest?** Students flip five cards from their shuffled stacks as described in order to create the largest number possible. Once the game has ended, students stand, one at a time, and read their numbers aloud. Whoever reads the larger number remains standing. When all have read their numbers, one student should still be standing, the one with the largest number. It is possible that several students might have the same number. In this case, more than one student remains standing. This activity could be repeated with the goal of creating the smallest five-digit number. See Blackline Master I - 1.
Extension: Use chips for creating decimal numbers. Add a rule that one card is flipped at a time and must be placed and not moved.

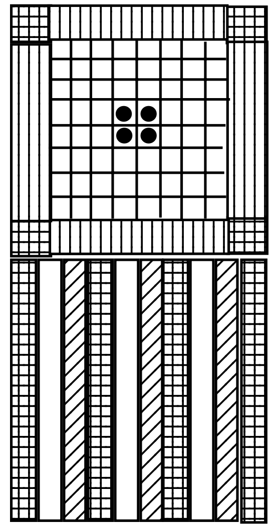
M. Have students play a version of "**Win a Dollar**". Small groups of students will need play money (pennies, dimes, and one dollar bills), and a 0-9 spinner guide or die. Students take turns rolling the die and taking the number shown in pennies. Thus, if 8 is rolled, the player gets 8 pennies. When students collect 10 pennies, they trade them for a dime, etc. until they have $1.00. The following rule might be added: on a turn, one can either roll and take pennies, or trade pennies for a dime or dimes for $1.00, but not both roll and trade. Each player must have the fewest number of coins possible at any given time. Money is real-life application of decimal numbers and a helpful model for many students. Students could be asked to draw pictures showing each move of a game.

N. Using a hundreds board and colored cubes, cover all multiples of 3. What part of the total grid is covered? Write your answer as _____ out of 100 parts, in words, as a decimal number, and as a fraction. Repeat the activity with multiples of 4, multiples of 6, multiples of 7, and multiples of 8. Notice the patterns made by the cubes. How can this pattern help you predict multiples beyond 100?

O. Make a display of sets of ten (10 people, 10 animals, 10 N.C. products). Within the set find two or more classifications (4 children and 6 adults). Write the subsets in words, decimals, and fractions. (Four-tenths are children. In this group 0.4 are children. When we classified the people, $\frac{4}{10}$ were children.)

Notes and textbook references

P. Give each student a copy of the Blackline Master I - 8. On the first grid ask students to color a stripe pattern using only two colors. Stripes may be narrow or wide. (Use a hundred board and connecting cubes for a concrete experience.) Write the decimal and fraction which shows how much of the total is covered by each color. Ask students what they might write if they were to divide the unit into 10 parts horizontally as well as vertically.
Directions for the other grids might include ....

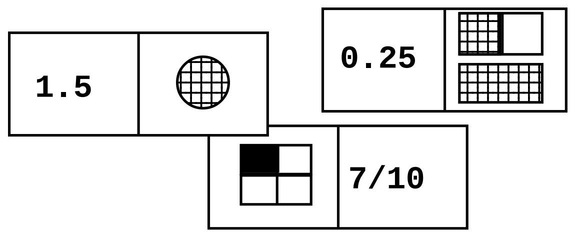


- Color your initials. Write the decimal and the fraction to show what part of the unit is your first initial, your last initial, the total uncolored, the total colored. What do you notice about the totals together?
- Color a pattern that might have been used by North Carolina native Americans. Tell about each color you have used by writing the decimals and fractions.
- Color a symmetrical design. Write the fraction and decimal numbers which tell about each color.
- Suppose you asked 10 people to name a single digit number. What do you think that would be? Make a design with that number in the center and some decoration in the corners. Write the decimals and fractions to tell about the colors you used. Make a graphic display to show everyone's numbers.
- Make a picture of a North Carolina product inside a frame. Write the appropriate decimals and fractions to tell how much of the total grid is not colored, how much is the frame and how much is the product.
- Make a design that is 0.28 red, 0.14 blue, and 0.30 yellow. How much of the total is not colored? Write the answer in three ways-words, decimal, fraction.
- Make a design in which 0.60 is colored and 0.40 is not colored. How much of the total is each color you used?

Q. Have the students cut centimeter graph paper (Blackline Master I - 25) into 10 x 10 squares. Using the 100 squares, demonstrate a variety of fractions and their decimal equivalents by shading in the appropriate number of squares. Use fractions that are seen most commonly, such as one-half, three-fourths, etc. Write the equivalents on the back of the square. For example, $\frac{1}{4} = 0.25$.

Notes and textbook references

R. Have students work in groups to make fraction/decimal dominoes. One end of the domino would have a picture, the other end would have the fraction or decimal notation. An important part of the task is to figure out what they will make. Allow students to sketch the total set in miniature before making the finished dominoes on heavy paper (old file folders, for example). Let them decide whether every set must have the same dominoes, what would constitute a set, how many matches they want, etc.

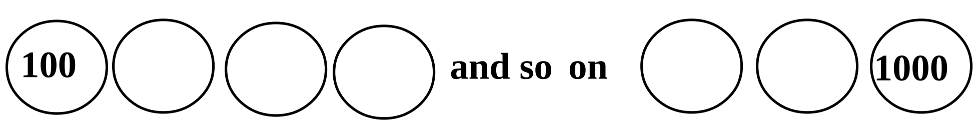


S. Give students a square of graph paper that is 4 x 4. Have them color a pattern that has an area of 8 square units. How many different patterns can be created? (This is also an opportunity to talk about what fractional part or percent has been colored.) This could be extended by changing the number of units to be colored and the size of the original square. Students could also be asked to group the designs according to some criteria they choose. This sorting activity could be repeated many times using different criteria.

See Blackline Masters I - 9 and I - 10 for gameboards and spinners to play this game.

T. Play **Line Up**! Each student draws a path of fifteen circles, placing the number 100 in the first circle and 1,000 in the last circle. Partners take turns rolling 3 number cubes. Two cubes should be regular 1 - 6 dice and the third cube numbered 4 - 9. When the three cubes are rolled, the player decides which cube represents the ones, which represents the tens, and which represents the hundreds. The resulting number is placed in one circle on the path. Subsequent numbers must be placed in the path so that they remain in a series from least to greatest. The player who is able to fill in the last number wins. **Problem solving through the game:** Does it matter who goes first? Are there some early strategies in deciding what numbers to form and where to place them that will help you win?

Variation: Each player has his or her own set of 15 circles instead of the players using the same circles.

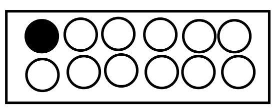


Notes and textbook references

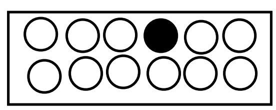
U. In groups of two to five, have the students discuss how U.S. coins are related to fractions and decimals. Then have them make a chart similar to the one below in order to make comparisons.

Coins	Decimal fraction of a dollar	Common fraction of a dollar
1 penny	0.01	1/100
2 pennies	0.02	2/100
3 pennies	0.03	3/100
*		
*		
*		
2 dimes	0.20= 0.2	20/100=2/10=1/5
*		
*		
*		
1 quarter	0.25	25/100=1/4

V. Give each pair of students an egg carton and 12 two-sided counters (or 12 connecting cubes or multi-link cubes). Ask the students to represent one-twelfth by placing one counter in an egg slot.



or



Compare the students' egg cartons to show the many different ways to model one-twelfth. Repeat the activity with other fractions like $\frac{1}{6}$, $\frac{5}{12}$, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{1}{4}$, etc.

W. Prepare three sets of cards: one set with numbers 0 - 9, another with ten different familiar fractions, and a third set with decimal numbers. Shuffle decks together and have students draw 3 cards, arranging them from least to greatest (or greatest to least). See Blackline Masters I - 11 through I - 13.

X. **Concentration** (Blackline masters I - 29 through I - 34) and a modified **Bingo** are both good ways to become familiar with decimal numbers. See Blackline Masters I - 14 through I - 20. The **Concentration** cards can be used in other ways. Deal five cards and ask students to order the numbers. They can also be used in the game of **War**.

Notes and textbook references

Y. Students, working in groups, use one set of decimal models and decimal number cards that were previously made to play an equality game called "**Turn Over**." Each player receives 6 decimal models to place face-up in two rows. The cards are shuffled and placed in a stack face-down where all players can reach. A player turns over the first card and shows it to the entire group. Anyone who has a decimal model that is equivalent to the one who turns over their 6 decimal models wins the game.

Z. Use the **Big Four** game as described previously in 1.01 b) - D. Ask students to work in small groups to build the "winning number" with paper base ten materials and then draw pictures of this construction. A combination of regular base ten blocks and student-made materials will probably be needed because of the possibility of large numbers. Students can use grid paper to cut units, longs or tens, flats or hundreds, and construct cubes that are 10 by 10 by 10 to represent thousands.

Play this game by establishing a "target number" and setting the goal of creating a number as close as possible to this target.

Add the use of a calculator to this game as follows: students flip four cards to create their **Big Four**; they enter this number into a calculator; then without clearing the display, students must change this number to equal that of the highest number from the game. Students who have created the largest number during the **Big Four** game could be asked to enter it on the calculator and change the display to equal the smallest number created by some other student. This game could also be extended by having students flip more cards to create larger numbers, 5-digit or 6-digit.

Challenge students to build a model showing 100,000. How much larger is 999,999? Students might be challenged to write a book titled <u>How Much is a Hundred Thousand?</u> For example, did you know that 100,000 seconds is almost 28 hours, or almost 4 hours more than 1 day, or 1.16 days? Did you know that 1,000,000 days is almost 274 years? My, how time flies!

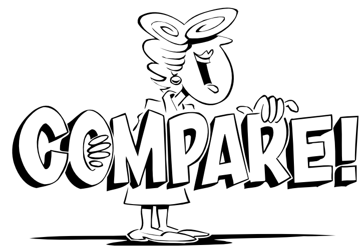
AA. **Buckets and Bowls** You will need 3x5 index cards and small buckets or plastic bowls. Label each container and place them in order from left to right as **Near 0**, **About $\frac{1}{2}$**, and **Close to 1.**

Notes and textbook references

Students should work with partners to make Decimal Cards. Cut paper or use 3 x 5 index cards. See Blackline Masters I -14 through I -16. Suggestion for numbers to place on cards

0.90	0.9	0.49	0.83	0.07	0.09	0.18	0.22	0.55
1.05	0.88	0.4	0.21	0.04	0.59	0.02	1.06	0.75

Sort your cards into three groups. Near 0, About $\frac{1}{2}$, and Close to 1



Make 3 new cards for each group.

Put all of your cards in order by size.

Extensions: Do a similar activity but use fraction cards. (Blackline Masters I - 29 through I - 34). Do a similar activity but mix decimal and fraction cards.

Notes and textbook references

In 1584, Simon Stevin (1548-1620) stated rules for computing decimals and gave practical applications for them in La Disme.

d.) Make estimates of rational numbers in appropriate situations.

A. Make three circles using hula hoops, string, yarn, etc. Label the circles with 0, 1, and 2. Give each student a card with a fraction (or decimal) number between 0 and 2 on it. Students are to decide which number (0, 1 or 2) the fraction (or decimal) is closest to and place their cards in the correct circle. Students should explain why they placed their cards as they did.

B. Use the same activity as shown in 1.01d) - A, but this time use a clothesline with 0, 1, and 2 hanging in correct order. Have the students order the cards and hang their cards in the appropriate place. Discuss the reasonableness of their choices.

C. Use a clothesline with several whole numbers between 0 and 10 hanging on it. Give the students cards with mixed fractions and decimals. Have them hang their cards appropriately on the number line (clothesline). As a class, estimate the placement of the numbers.

D. Have each student draw 10 lines on a sheet of blank paper. Then, give the paper to a partner and have the students estimate the length of the lines. The students will use a ruler and measure the lengths of the lines. They can then use the ruler to show which whole inch (or centimeter) the measurement is closest to.

E. Have students estimate and justify answers to questions such as:

- How many jelly beans would fit into a pint jar?
- How many Fridays are there during the current year?
- How many brown-eyed people are in the whole school?
- Is the number of Fridays always the same per year?
- How many times can you chew a bite of food?
- How many pennies do most adults carry in their pockets?
- How many pages are in the dictionary?
- How many times can you pat your feet in a minute, etc.?

Notes and textbook references

F. Estimate, then calculate how many days it would take to travel from Raleigh to your hometown if you traveled 10 miles forward and 5 miles in reverse each day.

G. Divide the class into groups to discuss reasonable strategies for solving the following problems:

- What size box would you need to mail a skateboard?
- How would you measure the distance around an elephant?
- How much paper would you need to wrap your friend's birthday present?
- How would you find the weight of a bumblebee?
- How much water could your bathtub hold without overflowing?

Encourage students to create other thought-provoking problems.

H. Estimate how long it would take to walk from your hometown to some selected destination. Before giving their estimations, students will need to have some general idea of how long it takes to walk a mile. The teacher may decide to have the students compute the exact time. If so, it would be necessary to measure the students' walking pace.

I. Which prize would you prefer, a million dollar jackpot or collecting $0.01 the first day and doubling the amount each day for 30 days? (Day 1 = $0.01, day 2 = $0.02, day 3 = $0.04, etc.)

J. Students are shown a glass jar filled with about 200 marbles and are asked to estimate the number in the jar. In small groups, they discuss various approaches to the problem and the strategies they can use. They settle on a strategy to share with the class along with the estimate that resulted.

Notes and textbook references

K. Assign a specified amount of money to each student, then display on the board or overhead pictures of items and their prices. (Give a wide range of prices for the items.) Students take turns representing their team and naming items they think will be close to their target amount without going over. They should not do the actual arithmetic until time is called and the students closest to the total are recognized. Allow students to use calculators at this point. This activity should be repeated many times with many different target amounts to spend and different items to buy

L. Use menus from local restaurants or have the class make one. Give a fixed amount such as $5.00 for lunch and have students select from the menu. Estimate the total meal cost. Estimates can be checked with a calculator. (You can introduce tax tables rather than compute sales tax.)

M. Give students questions with computations where they don't have the time to compute but must make judgments based on estimation. Later, give students time to check and verbally justify their answers to a partner. This is a good activity for the overhead. For example, "Would 250 be a good estimate for 24 x 9?" or "If I add $4.25, $8.69, and $6.85, will my total be more or less than $20.00?"

N. Put a chart with populations of North Carolina cities on the overhead. Ask students to estimate which two or three cities together would have populations of a certain amount. Move quickly so that students learn to estimate rather than take time to compute.

O. When you use pages of drill in the textbook, give students a variety of assignments which encourage estimation and logical thinking. For example:

- Work only the problems which will have odd numbers as a product.
- Work only the problems which will have products greater than 100 (or another appropriate number).
- Work only the problems whose answers are between 500 and 900.
- Work only the problems in which the number in the tens place of the answer will be less than 6.
- Work only the problems which will have products less than 100.

Notes and textbook references

P. Examine the results (with a calculator) of multiplying numbers by 99 or 98. When a pattern emerges have students form strategies for mentally multiplying by these numbers. Share other mental math strategies.

Q. Students use base ten blocks to help them decide how many blocks there would be in each group if they divided 123 blocks among three people. The students describe how they used the blocks to help them solve the problem and compare their solutions and solution strategies.

R. Record how much paper is used by one student in a typical day. How much would that be for your class? Your grade level? Your school? Can you estimate the paper use per week for your building or your district? Is paper use the same in grade 1 and grade 5? Why?

S. How many times do you blink in a minute? In an hour, a day, a week? How long is a blink? While going home from school, how long are your eyes closed?

T. The band has 103 students in it. They line up in nine rows. How many students are in each row? The students' responses might indicate for example, that there should be about 10 students in each row.

U. Students regularly estimate in situations involving classroom routines. For example, they may estimate the total amount of money that will be collected from the students who are buying lunch on Pizza Day or the number of school buses that will be needed to take the whole fourth grade on the class field trip.

V. Use rounding in a flexible manner rather than according to out-of-context rules. In a grocery store, for example, when a person wants to be sure there is enough money to pay for items that cost $1.89, $2.95, and $4.45, the best strategy may be to round each price up to the next dollar. In this case then, the actual sum of the prices is definitely less than $10.00.

Notes and textbook references

1.02 Develop fluency with multiplication and division:

a.) Two-digit by two-digit multiplication (larger numbers with calculator).

A. **Base Ten Blocks** can be used in a similar way as arrays. Use Base Ten Blocks or paper models to model 2 digit by 2 digit multiplication. This may help students who are having difficulty and need more practice.

Students work in groups to solve problems such as 4 x 38 using base ten blocks. Present a problem such as 4 x 38. Ask students to describe what they see when thinking about 4 x 38. Begin by having students build one group of 38 using the fewest blocks possible.

Remind students, "You need 4 groups of 38. How might you do this?" *(Most likely students will add 3 more groups of 38. Some may trade in blocks. If you notice trading, then the next step is to have groups describe the blocks they used. Begin with a group who used 4 groups of 3 tens and 8 ones and next to a group who traded in blocks.)*

Connect student's ideas to the partial product algorithm.

```
  38   Provide opportunities for students to connect the model to the
x  4            algorithm.
 120
  32
 152
```

B. **Same Products** Give students a problem such as:

24 x 18 or 18 x 24

Write a problem that has the same answer. Possible solutions include:

48 x 9 , 12 x 36 , 6 x 72

Explain your reasoning. Prove you are correct by solving the problem.

New problem: 25 x 60

Write a problem that has the same product: 50 x 30 100 x 15

Explain your reasoning. Prove you are correct by solving the problem.

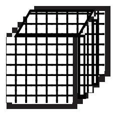
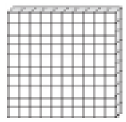
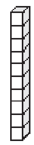
Some other problems to try:

84 x 60 (42 x 120 21 x 240)

75 x 60 (150 x 30 300 x 15)

C. **Base Ten Blocks** can be used to develop understanding of multiplying a large number by 10.

Have students use base ten blocks to show 654. Ask students to next create what the product will be if you multiply 654 x 10.

				
654		6 flats	5 rods	4 cubes
6540	6 blocks	5 flats	4 rods	0 cubes

Students should notice that multiplying by 10 moves the amount in each column over a column to the left. Student boards should show 6 one thousand cubes, 5 hundred flats, 4 tens rods and zero units. Students need opportunities to practice and discuss several problems. Elicit discussions as to why there is a zero in the units place.

Ask what would happen to 654 if you multiplied by 20? You might give students grid paper to help keep track of numbers. Give students opportunities to test ideas.

Notes and textbook references

Factors of Benchmark Numbers may be regarded as an enrichment activity.

Notes and textbook references

Some students may have difficulty transferring the division algorithm to calculator use. Be sure they are entering the dividend first. Both $6\overline{)18}$ and 18 ÷ 6 are read "eighteen divided by 6."

b.) Up to three-digit by two-digit division (larger numbers with calculators).

A. Use real-world situations in which students can draw, act out, and/or explain the process of division. Here are three examples which students can model:

1. The class has one large bag of pretzel sticks with (put actual number in the bag here) pretzels in it. There are 26 (use the number of students in your class) students. How many pretzel sticks will each student get if they are shared equally?

2. The students went on a lost pencil search around the school. They collected 83 pencils. How many groups of 15 pencils can they rubberband together?

3. At the end of school the classroom supplementary books were stored in the back of the library. There were 168 books. If the 12 students helping the teacher each carried the same number of books, how many books did they carry? Were there any left for the teacher to carry? Explain the strategy used to solve this problem.

B. Two columns of numbers are written on the board. Column I contains 2-digit multiples of 10, column II contains 3-digit multiples of ten. Students are divided into two teams. The teacher points to two numbers, one from each column. A student from team 1 will have five seconds to estimate the quotient of the number of column II divided by the number in column I. The teacher will alternate giving problems to each team. Each team starts out with 50 points. If the student's answer is correct, the team gets 10 points. If the student's answer is incorrect, five points are deducted from the team's score.

C. Base ten materials are only one model which can be used to explain the division process. Ask students to create and explain their own models. Brainstorm a list of possibilities such as money, arrays, collections of objects, and connections to multiplication. Have students share their models with each other.

Notes and textbook references

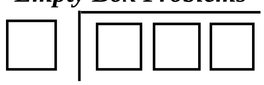
D. Have students solve several division problems, most of which have remainders, whether using the paper and pencil algorithm or the **INT ÷** key on the Math Explorer™ Calculator. Then have them write the remainder as a fraction. If the fraction is "messy," have them round it to a more familiar fraction (i.e., 9/21 is close to 10/20 or 1/2). Relate the rounded fractions to decimal equivalents. Discuss the patterns and relationships which they see.

Problem	Quotient and Remainder	Quotient and Fraction Remainder	Decimal Estimate	Calculator Solution
$14\overline{)805}$				
$16\overline{)875}$				
$32\overline{)176}$				
$12\overline{)3256}$				

E. Model this process using base ten blocks. Here is an example of how to proceed. Consider the problem 127 ÷ 6. Build 127 with base ten blocks using one flat (100), two longs (10) and seven cubes (ones). The task is to divide this number, 127, into six groups or to subtract six over and over until it's no longer possible. In order to divide the flat into six groups, it must first be traded for ten longs. These ten longs along with the two longs create 12. These 12 longs are distributed into six piles with two in each. Now the seven cubes are distributed so that one goes in each pile and there is one left. This model shows how 127 is divided into six groups, with 21 in each group and one left over. Ask students to explain how to model subtracting six continually until it is no longer possible.

Notes and textbook references

Empty Box Problems

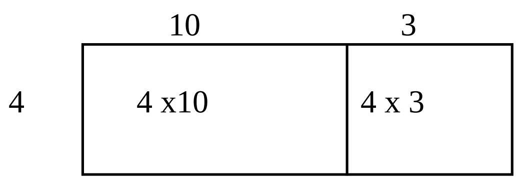


F. Create a variety of "empty box problems" for students. Begin by drawing four boxes as shown. Ask students to draw the same box arrangement on paper. Explain that the goal is to write the numbers you call out in the boxes in such a way that the greatest quotient results. Once a number is placed, it cannot be moved. Roll a die with ten faces (or use a spinner) numbered 0 through 9. These are decahedrons and are available through educational supply catalogues. Announce the resulting number. Students write this number in a box. Repeat this three more times. Ask students to find their quotients and raise their hands if they think they have the largest quotient in the class. Compare answers from students. Ask students to use the box arrangement and the numbers generated to find the quotient closest to 200. Discuss strategies students use for deciding where to place a number rolled. Where is the "best" place to write a 2 when trying to create the largest possible quotient? Explain your thinking. Is there any place that a zero cannot be placed? Give students the option of multiplying a number rolled by 10 and using it as the divisor. For example, if 2 is rolled, a student may decide to multiply it by 10 and have 20 as a divisor. Divide students into small groups and give each group a die or spinner. Let them play by rolling their own numbers. Students take turns rolling but everyone in that group places the number rolled.

c.) Strategies for multiplying and dividing numbers.

A. This activity uses arrays to help children see how a multiplication problem can be solved using facts they know. Organize the students in groups of two to four. Each group needs several pieces of centimeter graph paper (Blackline Master I - 25).
After groups have created several new arrays and written the sentences for them, have them share the arrays they have found. Discuss how facts you know like 6 x 6 and 6 x 7 can be used to solve multiplication problems you might not know as readily like 6 x 13.

B. When students have had experience building arrays with the base ten blocks, introduce them to a pictorial representation of the array. With larger numbers, it is inefficient to draw an array showing each part, so tell the children this is an open array, representing 4 x 13:



Ask the students if the array showed all the parts, how many squares would be in each part. You might record it as you recorded the base 10 arrays:

4 x 13 = 4 x 10 + 4 x 3

= 40 + 12

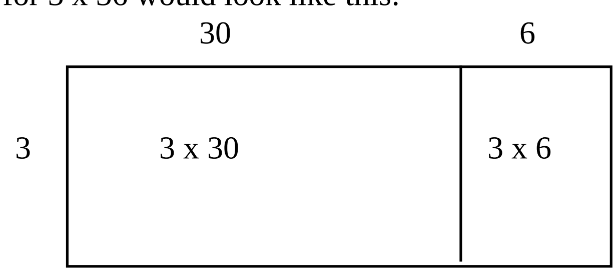
Or you may want to show students this way to record the partial products. (Be sure they know the term *partial product*.)

4 x10 = 40
4 x 3 = 12
4 x 13 = 52

Have students draw the open array for other problems such as

4 x 17   5 x 19   3 x 28   7 x 23   3 x 36

The array for 3 x 36 would look like this:



3 x 30 = 90
3 x 6 = 18
3 x 36 = 108

Notes and textbook references

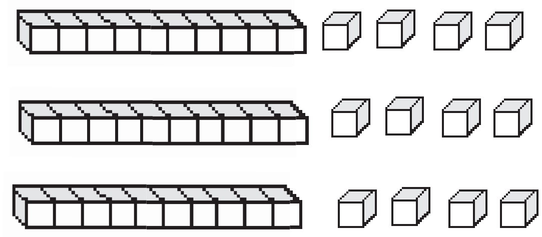
The array model is a powerful tool for representing multiplication. From the simple arrays that match the basic multiplication facts to arrays for larger numbers, they give children a visual model to help them understand the meaning and process of multiplication. Arrays can be useful in showing how numbers can be composed and decomposed to create new numbers using known facts.

Notes and textbook references

Research shows that children who are taught procedures or algorithms too soon lose their number sense and spend their time trying to remember the steps of the procedure rather than using their number sense and understanding of the meaning of the operation to evaluate answers to see if they are reasonable. This research seems to indicate that it is good for children to develop their own strategies for these operations before we insist that they learn the algorithms that many think are "the way" to multiply or divide. Don't be afraid to pose a problem and ask your children for the different ways that they might solve it. Some will have been taught procedures at home, others will have wonderful mental strategies which we should encourage. It is important that children learn to be flexible and to know that there is not just one right way to solve a mathematics problem. There may be one right answer, but there are almost always multiple ways to get to that right answer. We must honor the understandings and number sense that they have.

C. For these activities, students should work in pairs. Each pair will need a bag of base ten blocks with a minimum of 2 hundreds, 20 tens, and 30 ones, and paper for recording. Have extras available if possible.

Ask the pairs of children to build 14 three times. Model how to do this on the overhead with transparent base 10 blocks if you have them. It is important that the model be in a rectangular formation like this:



On the recording sheet, have the students write:

3 x 14 = _______ + _______

Tell them that using the "magic" number 10 is very helpful in solving problems like this one. Lead them to see the three 14's as sets of tens and ones, through a conversation such as this:

Ask: "How many tens do you have?" (3)

Record like this: 3 x 14 = 3 x10 + _______

Then ask: "How many groups of 4 ones do you have?" (3)

Record: 3 x 14 = 3 x10 + 3 x 4

The children can easily solve these parts, recording like this:

3 x14 = 3 x10 + 3 x 4

[ ] = 30 + 12

Ask: "What is the total?", now easily figured out, (42), and have the children record the answer in the box:

3 x 14 = 3 x10 + 3 x 4

= 30 + 12

You may want to have the students make the appropriate trades of ones for tens to clearly show 42, but this is not necessary.

Notes and textbook references

Continue using this procedure for several more problems such as the following:

4 x 21   3 x 24   7 x 22   6 x 13   4 x 35*

Note that the array for 4 x 21 would be four rows like this, keeping the rectangular formation:

*After solving 4 x 35, ask the children if they think that 3 x 45 would give the same product as 4 x 35, then build the model and record the solution for 3 x 45.
(It does not. 4 x 35= 140, 3 x 45 = 135. Ask the children why they do not have the same solution. They should see that 4 x 30 = 3 x 40,
but 4 x 5 > 3 x 5.)

After you think that the students understand the process and have connected the model to the recording, have them try the following problems without building the model. (They won’t have enough base ten blocks to represent these combinations.)

6 x 38   9 x 42   8 x 76

Discuss the efficiency of using combinations of ten in solving multiplication problems.

Give the students lots of experience with this kind of model in problem situations such as the following:

A peach orchard owner wanted to know how many trees he would have if he planted 9 rows with 23 trees in each row.

Jim’s gas station sold 18 gallons to each of 8 people on Thursday. How many gallons was that?

The stock boy at Frannie’s Grocery put 34 jars of jams and jellies on each of 6 shelves. How many jars did he shelve?

Notes and textbook references

When multiplying a two-digit number times a two digit number, students need a model that helps them visualize and keep track of the numbers.

D. A larger open array can be used to represent multi-digit multiplication. For example, give the students the problem:
Mrs. Bing has planted 27 rows of cherry trees with 42 trees in each row.
One representation of this problem (27 x 42) would be this open array:

	20	7
40	40 x 20	40 x 7
2	2 x 20	2 x 7

40 x 20 = 800
40 x 7 = 280
2 x 20 = 40
2 x 7 = 14
1134

For larger numbers, the array increases by the necessary number of partial products:

345 x 48 would be represented thus:

	300	40	5
40			
8			

E. To begin to solve multiplication products more symbolically, students can decompose the larger numbers into expanded forms, such as the following:

27 = 20 + 7	367 = 300 + 60 + 7	49 = 40 + 9
x 4 x 4	x 5 x 5	x 63 = 60 + 3
28 (4 x 7)	35 (5 x 7)	27 (3 x 9)
80 (4 x 20)	300 (5 x 60)	120 (3 x 40)
108	1500 (5 x 300)	540 (60 x 9)
	1835	2400 (60 x 40)
		3147

It can be very helpful for students to write the steps out to the side as shown above. It helps them not forget any of the subproducts and reminds them when they are multiplying by tens or hundreds so that they more easily remember the necessary zeros.

Notes and textbook references

As students begin to use the “standard” compact multiplication algorithm, it is very helpful to compare the compact subproducts with these expanded subproducts. It is from a progression of experiences like these that the formal compact multiplication algorithm begins to make sense to children. (See 1.02c) F and G following. When the children are comfortable with finding all the partial products, have them try to find them without writing the numbers in expanded form. Not all the children will be ready for this at the same time.

F. Moving from more accessible formats like those presented in 1.02c) B-E to the compact multiplication algorithm needs a careful progression. Use these ideas to help children see how the whole list of partial products and a compact solution match. The amount of time you spend on this will depend on the mastery of 1-digit by 2-digit numbers the children have brought with them from third grade.

Solve 36 x 7 writing all the partial products. Then rewrite the problem next to the first and talk it through as follows:

```
  36        36
x  7      x  7
  42       252
 210
 252
```

“Seven times six is 42. That is 4 tens and 2 ones. I write the 2 in the ones place and remember the 4 tens. I may even make a note of it.
“Seven times 30 or 3 tens is 21 tens plus the 4 tens I remembered is 25 tens. I write 5 in the tens place and 2 in the hundreds place.”
Point out that 252 is the sum of the two partial products 42 and 210. The 4 tens in 42 and the 1 ten in 210 were added to get the 5 tens in the product in both problems.
Give the students as much experience with this process as needed.

Move on to one-digit by 3-digit multiplication, using this problem:

```
  536       536
x   7     x   7
   52      3752
  210
 3500
 3752
```

Discuss these solutions in the same way. Be sure the children see the relationship between the three partial products and the way they add in the regrouped tens and hundreds in the compact form.

Notes and textbook references

G. When the students have had lots of practice multiplying by one digit, move to multiplying by two digits. Refer back to the array model for a 2-digit by 2-digit number from 1.02c) D and the expanded form from 1.02c) E. Once they are comfortable writing all the subproducts without decomposing the factors into expanded form, show them the compact form, by comparing it to writing out all the subproducts. Use a discussion like the following:

36	36	
x 47	x 47	
42	252	(7 x 36)
210	1440	(40 or 4 tens x 36)
240	1692	
1200		
1692		

"First we multiply 7 x 36." See the discussion in 1.02c) F for how to find the compact subproduct.
"Next multiply 40 or 4 tens times 36. Four tens times 6 is 24 tens. Twenty-four tens is 2 hundreds and 4 tens. I write 4 in the tens place and 0 in the ones place (since I'm multiplying by a number of tens, there will never be anything but 0 in the ones place in this second subproduct). I remember 2 hundreds."
"Four tens times 3 tens is 12 hundreds plus the 2 hundreds I remembered is 14 hundreds. I write 4 in the hundreds place and 1 in the next place, the thousands place."
"Add the two subproducts to get 1,692, which is the product of 47 and 36.

Some other points should be made using appropriate examples such as the following:

Sometimes there is something to "remember" or regroup and sometimes there is not.

A zero subproduct may be written down to avoid confusion. *

It is necessary to include zeros that occur in other place in the subproducts.

22	79	30	34
x24	x60	x35	x56
88	0	150	204
440	4740	900	1700
528	4740	1050	1904

Talk through each of these kinds of problems. For as long as you think it is helpful, contrast the compact version with the longer version.
*If you think that students understand what they are doing without writing the zero subproduct, let them solve without writing the zero subproduct.

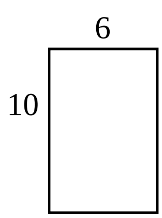
Notes and textbook references

H. **Doubling and Halving** is a strategy that is sometimes helpful for solving multiplication problems. For example, to solve the problem 15 x 80, you might double the 15 to 30 and half the 80 to 40 and then solve 30 x 40, a simpler problem. To get the students to think about this, pose a problem like 15 x 80 and ask them what mental strategies they might use to solve it. If no one suggests doubling and halving, suggest it yourself. Show several other problems such as these:

17 x 42 24 x 14 40 x 18 35 x 26 11 x 16

Ask the students which problems might be easier to solve by doubling and halving.

I. **Using Big Arrays and Small Arrays** Draw an array or cut and label an array from grid paper. In the beginning require students to keep one dimension and decompose the other dimension. This enables students to see the parts. Differentiate instruction by removing this restriction for students who understand the ideas.



Draw 2 more arrays that together will equal the size of your first array. (Students might draw a (4 x 6) + (6 x 6) or (2 x 10) + (4 x 10)

Students need many opportunities dividing 100 grids into 4 arrays and writing multiplication equations to label dimensions of arrays.
An array is a geometric picture of and multiplication can be used to build understanding of the commutative and distributive properties of multiplication.

<u>Extension</u>: Students can find as many ways as possible to split an array into two or more arrays. They might search for combinations of arrays that cover the same area.

<u>Extension</u>: Move to two digit by two-digit arrays.

Notes and textbook references

J. **Factors of Benchmark Numbers** Give students snap cubes. Ask students to find all the factors of 10. Introduce Factor Pairs by eliciting an answer to the question, "How do you know you have all the factors. Teacher should list factors of ten as well as factor pairs of ten.

Factors of 10 1, 2, 5, 10
Factor Pairs of 10 1 & 10 2 & 5

Ask students to create a chart and list the factors of 10, of 100, of 1000. Identify patters that occur.

Students work with partners or in small groups.
Are factors of 10 also factors of 100? Find all the factors of 100.
How do you know you have them all?

Are all the factors of 100 also factors of 1000?
Convince someone that you have found all of the factors.

Have students share strategies. Did any patterns emerge?

Twenty is a multiple of ten. Are the factors of 10 also factors of 20? Are factors of 20 also factors of 40?

Are the factors of 9 also factors of a multiple of 9? How do you know?

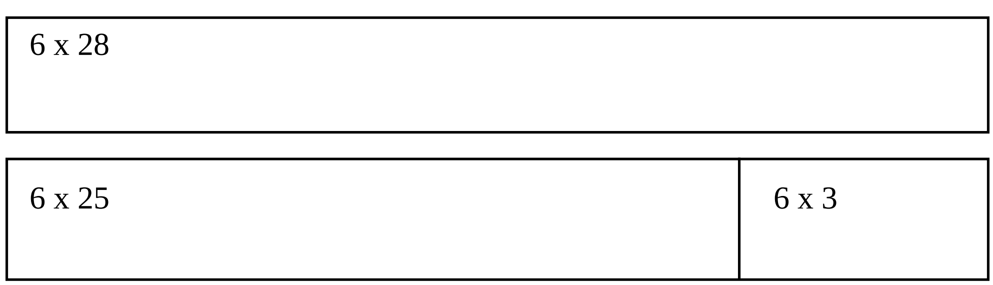
The Distributive Property states that an operation acts on a group in the same way it acts on each element of the group. See the <u>Glossary for Parents and Teachers</u> for examples. Go to Instructional Resources at: community.learnnc.org/ dpi/math/.

K. Use the **Distributive Property** as a strategy for making sense of multiplication of large numbers. Pose the problem 6 x 28. Ask students how they might use ideas of place value to break apart the 26 to make it easy to use multiplication.

6 x 28 = (6 x 25) + (6 x 3) The 28 has been decomposed as 25 +3.ß

Using centimeter grid paper (Blackline Master I - 25), have students draw a rectangle that shows 6 rows of 28 squares.

Underneath, have students draw two rectangles, one showing 6 rows of 25 squares and another showing 6 rows of 3 squares.



This model illustrates the idea that decomposing or breaking the rectangle anywhere doesn't change the number of squares.

Challenge students to find other ways to divide a 6 x 28 rectangle.
Be sure to label all the parts.

Now have students create a different rectangle model for 53 x 8 and show at least two different ways to divide the rectangle.

L. Make the Distributive Property explicit.

For example: 3 x 26,
Distribute the factor of 3 over the addends of 26.
(3 x 20) + (3 x 6) = 3 x 26

________ + _______ = 78

Solve several problems together and then pose problems for the students to solve with partners. Share strategies.

Ask students to use the distributive property to solve 6 x 36.

Give students problems such as 18 x 9. In this example, lead students to decompose the 18 as 10 + 8

(10 x 9) + (8 x 9) = 18 x 9

Two-digit x two-digit multiplication
Move to two-digit by two-digit examples. Begin with problems like 12 x 36. Students can easily decompose 12 into 10 + 2. You may ask them to decompose both factors. The teacher needs to model this process, while eliciting ideas of students.

Students will need lots of opportunities to practice this strategy through problem solving or story problems. Give students multiple opportunities to use this model.

Notes and textbook references

Students come to fourth grade with misconceptions of equality. Be sure to include equations where the values on each side of the equal sign are equivalent but no "answer" is given.

Notes and textbook references

M. Students might do some research to learn more about early methods for multiplication,
For many years the Egyptians solved multiplication by a method based on doubling. See http://faculty.ed.umuc.edu/~swalsh/Math%20Articles/EgyptMultiply.html
Russian peasants used halving and doubling to solve multiplication. See http://mathforum.org/dr.math/faq/faq.peasant.html for more.

Students might research to learn more about these methods from the past and solve problems using these the Egyptian method and the Russian peasant method.

N. **Story Problems**: Create problems that provide opportunities for students to apply understanding of 2-digit by 2-digit multiplication. Encourage use of arrays and break- apart-models.

Example:

Brad planted 14 rows of tomato plants. There were 23 plants in each row. Ask students to close their eyes and visualize the problem. Ask different students to describe what they see.

Ask students to solve the problem. Provide opportunities for students to share solution strategies. Encourage the use of arrays, the use of easier facts as well as versions of the traditional algorithm

Create problems that integrate to other strands of mathematics.

Mary bought 6 meters of ribbon. The cost of the ribbon was 25 cents for 100 centimeters. How much did Mary pay for ribbon?

Carlos and Robert built a dog pen for Robert's dog. The area of the dog pen is 23 square yards. Robert wants to know the area of the dog pen in square feet. Solve this problem for Robert.

Jasper's Work Schedule	Friday	Saturday
	8:30 a.m. – 2:30 p.m.	12:30 a.m. – 6:30 p.m.

Jasper worked 4 weekends each month for a year. How many hours did he work in a year?

Lucas counted 8 red hearts on one cupcake. If there are the same number of red hearts on each cupcake, how many red hearts are there on 3 dozen cupcakes?

The Morris family spends an average of $76.00 a month for pizza. If this continues, how much will the Morris family spend for pizza in a year?

The museum is celebrating its 12-year anniversary. The museum is open 359 days a year. How many days has the museum been open?

Give students opportunities to write their own problems. You might give them related equations such as:

12 x 24	288 ÷ 12 = 24
24 x 12	288 ÷ 24 = 12

In small groups, have students share their problems. Each small group might choose one person to share a problem with the whole class. As students read problems aloud, other members of the class will identify the equation used and explain their reasoning for choosing a specific equation.

O. Raging Rectangles

Raging Rectangles provides students with opportunities to explore arrays in a game format. Use number cubes and grid paper to play. The winner is the player who has the most squares colored at the end of the game. A gameboard is found in Blackline Master I - 26.

Notes and textbook references

Using number lines as models may help students build understandings of the relationship between multiplication and division.

Notes and textbook references

Connect to 5.03 b) Order of operations and the identity, commutative, associative, and distributive properties.

P. **Breaking Problems into Familiar Parts** Use multiplication facts you know to solve problems with larger numbers.

Visualize 12 groups of marbles with 14 marbles in each group.
You might think of 12 as 10 groups + 2 groups.

12 x 14 = (10 x 14) + (2 x14) =

__________ + _________ = __________________

Model with a variety of problems. Pose problems and have students give simpler facts that would help figure out the problem.

Suppose you had to solve 12 x 36. What facts do you know that can help you solve this problem?

Possible facts to present:		
	10 x 30	10 x 6
	2 x 30	2 x 6
	10 x 36	2 x 36

Elicit students' ideas and have them explain why certain facts would be useful.

When students have had sufficient experience, move to more complex problems:

Suppose you had 24 rows of chairs and there were 42 chairs in each row.
You might think of 24 rows as 10 rows + 10 rows + 4 rows

24 x 42 = (10 x 42) + (10 x 42) + (4 x 42)

__________ +_________ +_________ =__________________

If you know 2 x 42, how can this help you think of 20 x 42?

24 x 42 = (20 x 42) + (4 x 40) + (4 x 2) =

Extension: Connect to Geometry when finding the number of small cubes in a rectangular prism or cube. This is a three dimensional array. Students might use layers to count the number of small cubes in a cube or rectangular prism.

Q. Number lines

Notes and textbook references

Give students meter tapes that go to 100 centimeters or beyond. Students can make number lines using adding machine tape. Give students small markers such as numeration unit blocks.

Grasshoppers like to hop on a number line. Each grasshopper can only hop in equal jumps. This means if a grasshopper jumps by 3's, the grasshopper always jumps by 3's. Pose problems similar to the following:

A grasshopper that jumps by 3's jumped 25 times. Were did the grasshopper end?

A grasshopper that jumps by 12's jumped 8 times. Where did the grasshopper end?

A grasshopper jumped by 3's and landed on 48. How many jumps did it take? How do you know?

Is there another number the cricket can jump by and land exactly on 48? How do you know?

A grasshopper jumped by 4's and landed on 100. How many jumps did it take? How do you know?

Is there another number the cricket can jump by and land exactly on 100? How do you know?

For larger numbers, encourage students to draw open number lines:

100 — — — — — 250.

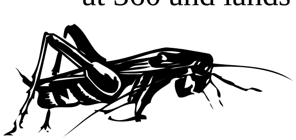
You might give students adding machine tape and thin markers to make numbers lines greater than 100.

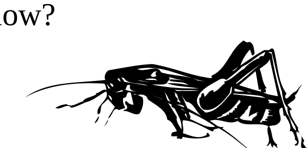
If a grasshopper that jumps by 10's begins at 120, how many jumps will it take to get to zero? How do you know?

What other numbers can the cricket jump by if the grasshopper begins at 120 and lands on exactly on zero? How do you know?

If a grasshopper that jumps by 10's begins at 360, how many jumps will it take to get to zero? How do you know?

What other numbers can the grasshopper jump by if the grasshopper begins at 360 and lands on exactly on zero? How do you know?





Notes and textbook references

Students should develop strategies and algorithms, which connect and support understandings of place value and the meaning of the operation.

R. Connecting to Traditional Algorithm

Use an array model for 8 x 32 to connect to the traditional algorithm.

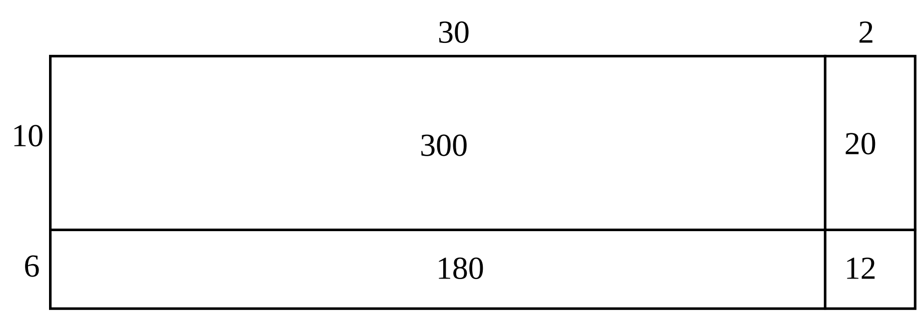
32 x 8

Elicit from student the connection from the array model to a partial product version of the algorithm.

```
  32
x  8
 240
+ 16
 256
```

```
  32
x 16
 180
  12
 300
+ 20
```

Partial products can be written in any order



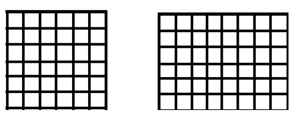
Notes and textbook references

S. This activity uses arrays to help children see how a multiplication problem can be solved using facts they know. Divide the class into groups of two to four. Each group needs several pieces of centimeter graph paper (Blackline Master I - 25). Give them these instructions:

1. Your group will need ten different graph-paper arrays. Cut them from the centimeter graph paper. You will need these arrays:

6×6	6×7	6×8	6×9	7×7
7×8	7×9	8×8	8×9	9×9

2. Take turns finding two arrays that can be joined together to make a larger array. When you have joined two arrays, write a sentence on your paper that tells how the arrays were joined and what new array you have. For example:



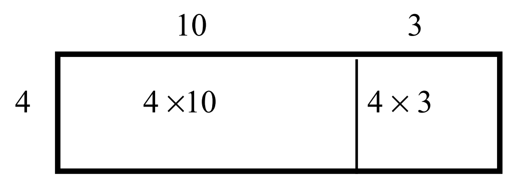
You would write $(6 \times 6) + (6 \times 8) = 6 \times 14$
Remember that your new array must also be a rectangle.

3. You can make 24 different pairs of these ten arrays by joining them to make larger arrays. See if you can find at least 10 of them.

After groups have created several new arrays and written the sentences for them, have them share the arrays they have found. Discuss how facts you know like 6×6 and 6×8 can be used to solve multiplication problems you might not know as readily like 6×14.

Notes and textbook references

T. After the class has had experience building arrays with the base 10 blocks, introduce them to a pictorial representation of the array. With larger numbers, it is inefficient to draw an array showing each part, so tell the children this is an open array, representing 4×13:



Ask the class if the array showed all the parts, how many squares would be in each part. You might record it as you recorded the base 10 arrays:

$$4 \times 13 = \underline{4 \times 10} + \underline{4 \times 3}$$

$$= 40 + 12$$

Or you may want to show the children this way to record the subproducts. (Be sure they know the term *subproduct*.)

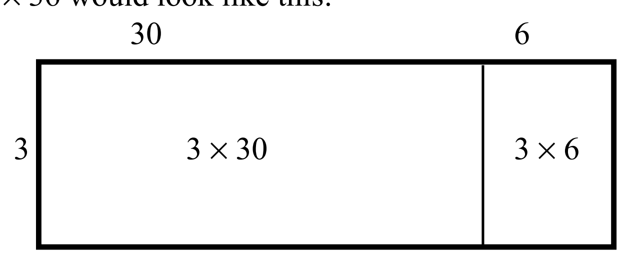
$4 \times 10 = 40$
$4 \times 3 = \underline{12}$
$4 \times 13 = 52$

Have the children draw the open array for other problems such as

4×17 5×19 3×28 7×23 3×36

The array for 3×36 would look like this:

$3 \times 30 = 90$
$3 \times 6 = \underline{18}$
$3 \times 36 = 108$



d.) Estimation of products and quotients in appropriate situations.

Notes and textbook references

A. Give each pair of students a copy of Blackline Master I - 44 Each student will choose a different colored crayon. The students will take turns choosing one number from group A and one number from group B and estimate their product. The student will then color the product on the playing board. The winner is the first one to get four in a row.

Students should develop strategies and algorithms, which connect and support understandings of place value and the meaning of the operation.

***Variation*:** Have the students change the numbers and create their own game boards.
Adapted from Estimation and Logical Reasoning by Dale Seymour Publications.

B. **Home on the Range** Put students in teams of 3 -5. Distribute the cards containing division problems from Blackline Master I - 45 among the teams of students. Have the teams take turns placing the problem cards under the appropriate range cards.

Extension: Students are still in teams of 3 - 5. Assign each team to one of the range cards. Direct the students to create problems to fit the range card. These could then be used in the game above.

C. Incorporate estimation in geometry by having students estimate products and quotients related to the area of rectangles. Ask students to estimate:

- the area when given the length and width
- the length when given the area and width
- the width when given the area and length

Have students share the different strategies they used to estimate. Discuss situations in which estimation is appropriate.

D. Play "Hit the Target" Blackline Master I - 46.

Notes and textbook references

e.)   Relationships between operations.

A. Draw a 3 by 4 array on the board. To give it a context, you might draw flowers to illustrate the a garden with 3 rows of flowers with 4 flowers in each row (or mushrooms, etc.) Ask volunteers to suggest multiplication or division sentences that are illustrated by the array. The following are the possibilities:

It is important that students see the sentences written in different ways so that they don't think the product in a multiplication sentence must be at the end, or the dividend in a division sentence must be at the beginning. They need the flexibility to write the sentences in different ways.

3 x 4 = 12	12 = 3 x 4
4 x 3 = 12	12 = 4 x 3
12 ÷ 3 = 4	4 = 12 ÷ 3
12 ÷ 4 = 3	3 = 12 ÷ 4

The students should suggest at least the first list. Suggest the sentences yourself in the second list if necessary. Emphasize where the total appears in each sentence. Then have them suggest sentences for an orchard with 7 rows of trees with 9 trees in each row. These sentences are the possibilities:

7 x 9 = 63	63 = 7 x 9
9 x 7 = 63	63 = 9 x 7
63 ÷ 7 = 9	9 = 63 ÷ 7
63 ÷ 9 = 7	7 = 63 ÷ 9

Give the children lots of practice with writing equivalent sentences for a given array.

B. After students have had practice writing equivalent sentences with numbers, include some sentences using letters rather than numbers to focus on where the total is in the sentence. For example, ask them to write equivalent sentences for this situation:

Jane ordered Z boxes of marbles to give to her friends.
Each box had Y marbles in it.
She had W marbles altogether.

Sentences include:	Z x Y = W	W = Z x Y
	Yx Z = W	W = Y x Z
	W ÷ Z = Y	Y = W ÷ Z
	W ÷ Y = Z	Z = W ÷ Y

This helps the students focus on where the total is in the sentence when the numbers don't give it away.

Notes and textbook references

C. Provide multiplication and division story problems for the students with a variety of possible sentences to represent the problem, some that fit and some that don't. They then choose sentences that fit the story. Here are a couple of examples:

Example 1. (total unknown)

At the dock on the Mississippi River, a steamboat picked up 20 bales of cotton. Each bale weighed 52 pounds. What was the total number of pounds of cotton the boat picked up? Choose from these sentences the ones that fit this story:

20 x 52 = ??	52 ÷ 20 = ??
?? = 52 x 20	?? x 20 = 52
?? ÷ 20 = 52	

(The sentences in the first column fit the story.)

Example 2. (one part – the number of groups – unknown)

One large steamboat had a large dining room for its many passengers. The boat could serve 96 passengers at one time at tables that each held 12 passengers. How many tables did the dining room have?

96 ÷ ?? = 12	?? = 96 ÷ 12
12 ÷ 96 = ??	96 ÷ 12 = ??
12 x 96 = ??	

D. Discuss how to solve number sentences like the following with missing factors. The students should see that they would be solved with a division sentence.

4 x ?? = 44	91 = ?? x 7

Equations like these can be illustrated with story problems such as the following. You might have the class come up with stories to fit the sentences.

<u>Story example 1</u>: Joseph had 4 boxes for his rock collection. He put the same number of rocks in each box. He had 44 rocks altogether in his collection.

<u>Story example 2</u>: Millie had 91 favors to give the friends who were coming to her birthday party. She was going to give each friend 7 favors. How many friends were coming to the party?

Notes and textbook references

E. Using stories and possible sentences like those listed in 1.02e) - C, have the class tell *why* some sentences match the story and some do not. More abstractly, give them a multiplication or division sentence and a list of possible equivalent sentences. Have them choose the true ones and tell why, then tell why the false ones are false. For example, given the sentence
M x Q = R, the sentence R ÷ Q = M is a true equivalent sentence because the factors and dividend are the same; but the sentence Q ÷ M = R does not, because in this sentence Q is the dividend, not the R which is the product in the original sentence. Or for the sentence

?? ÷ 21 = 7

7 x 21 = ?? is a true equivalent sentence, but 21 ÷ 7 = ?? is not because the dividend in the original sentence is the unknown but in this sentence the dividend is 21.

F. Related Facts for Multiplication and Division

These facts belong in a FACT FAMILY
3 x 6 = 18 6 x 3 = 18
18 ÷ 6 = 3 18 ÷ 3 = 6

Look carefully at each pair of related facts; write two other facts in the same family.

a. 6 x 9 =? and 54 ÷ 9 = ?
b. 4 x 7 = ? and 28 ÷ 7 = ?
c. 16 x 28 = ? and 448 ÷ 28 = ?
d. 27 x 4 = ? and 108 ÷ 27 = ?
e. 14 x 20 = ? and 280 ÷14 = ?

Students might choose a related equation and write a story problem to match the structure of the equation.

G. Related Facts for Multiplication and Division

These facts belong in a **Fact Family**

3 x 6 = 18 6 x 3 = 18
18 ÷ 6 = 3 18 ÷ 3 = 6

have students look carefully at each pair of related facts and write two other facts in the same family.

a. 6 x 9 =? and 54 ÷ 9 =
b. 4 x 7 = ? and 28 ÷ 7 =
c. 16 x 28 = ? and 448 ÷ 28 =
d. 27 x 4 = ? and 108 ÷ 27 =
e. 14 x 20 = ? and 280 ÷14 =

Students might choose a related equation and write a story problem to match the structure of the equation.

Notes and textbook references

The structure of problems fosters understanding of the relationship of multiplication and division.

H. The Distributive Property of Multiplication helps solve division problems.

Give students problems like 168 ÷ 14
Ask students how they can make this problem easier?

Many will know that 10 x 14 is 140.
Decomposing 168 into 140 + 28, gives students a way to make sense of this division problem.

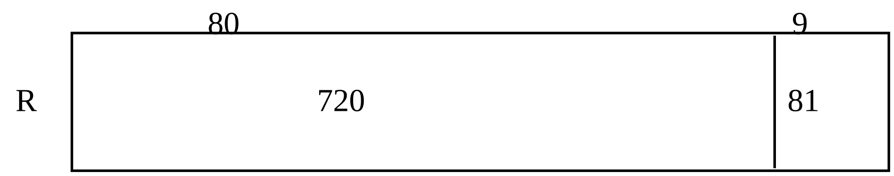
168 ÷14 = (140 ÷ 14) + (28 ÷ 14)
10 + 2
168 ÷14 = 12

Notes and textbook references

I. **Array Model**

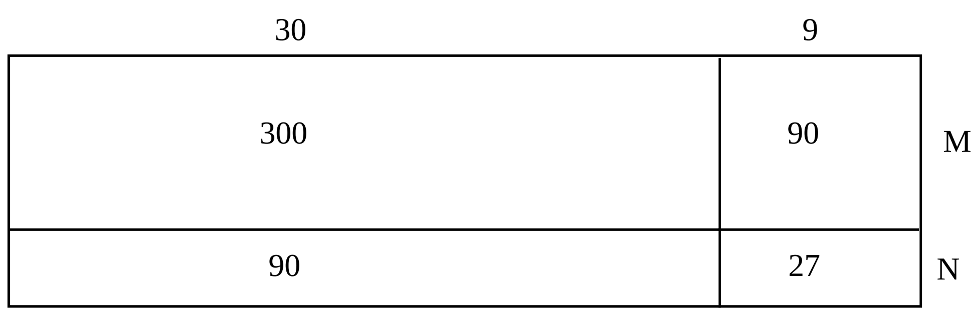
Continue to build on the array model, which builds ideas for students around the relationship of multiplication and division.

Give students arrays with missing factors.



What is the missing factor, R?
How do you know?
What multiplication equation is represented by the array?

What are the missing factors, M and N?
How do you know?
What multiplication equation is represented by this array?



1.03 Solve problems using models, diagrams, and reasoning about fractions and relationships among fractions involving halves, fourths, eighths, thirds, sixths, twelfths, fifths, tenths, hundredths, and mixed numbers.

Notes and textbook references

A. Present students with this scenario: You have been asked to design the floor plan for a new restaurant. Use the grid (Blackline Master I - 25) and shade in your floor plan. Label each section.

Suggestions:
Include 4 of the following areas.

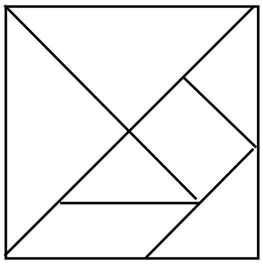
eating area	0. 5
dance floor	0. 25
restrooms	0. 03
entry area	0. 02
kitchen	0. 2

Write two fractions that are equivalent to the area of the eating area.
What two areas of the restaurant when added together equal three-fourths of the area?
The area of the dance floor is 0.25 Write two different fractions that are equivalent to this area.

Extension: Give students 100 grid paper. Students work with partners to design a gym, a room in the school, a room they would like in their house, a room for kids, etc.

Students should write questions for others to answer.
Students may use the model above or students might color in the grid and have other students figure out the area.

Notes and textbook references



B. Use your tangram shapes (Blackline Master II - 6) as an inspiration for problem-solving activities.

- If the square costs 80¢ what is the cost of the whole set? . . .of each piece?
- Suppose the middle-size triangle represents a one quart container. What will the other pieces hold?
- The whole puzzle is worth \$6.40. How much will each piece cost?

C. Anytime students are solving problems with fractions and their relationships, ask questions that promote using a variety of strategies and thinking about the best strategy to use.
Some questions you might ask:

- Try solving the following problems using drawings. Can you draw them in different ways?
- Which do you think is the most difficult? Why?
- How are the strategies alike/different?
- Can you use objects or manipulatives to explain/show your thinking?
- Why did you select that strategy? Is there a more efficient strategy you would like to try next?
- Is the same strategy most efficient in every problem?

Notes and textbook references

1.04 Develop fluency with addition and subtraction of non-negative rational numbers with like denominators, including decimal fractions through hundredths.

a.) Develop and analyze strategies for adding and subtracting numbers.

A. Using grids to add. (Blackline Master I - 27)
Jim walks 0.25 mile to school. After school, he walked 0.6 mile farther to his grandmother's house.

Did Jim walk more or less than 1/2 of a mile?

____

Use the grid to show the distance Jim walked.

How far did Jim walk? ______________-

Give students a page of grids and problems to add. Examples of problems are listed below.

Use grid to solve the following problems.

0.7 + 0.33 = _____________

0.79 + 0.04 = ____________

Use two or more grids to solve some problems. See Blackline Masters I - 5 or I - 27.

0.57 + 0.62 + 1.08 = ___________

Notes and textbook references

B. Give students a copy of this chart or a similar chart.

Shirley's Running Log

Day	**Mileage**	**Comments**
Monday	2.6	Started fast, then slowed down
Tuesday	2.75	Ran with mom
Wednesday	0.5	Too tired to run far
Thursday	2,4	Ran in a hilly area
Friday	3.6	Slow but steady
Saturday		
Sunday		

Shirley plans to run a 5.6 race next week. To prepare for the race, she wants to run **3 times that distance** this week. She has already run 4 days this week.

Complete the chart to show how far Shirley needs to run Saturday and Sunday to meet her goal. Show your work.

Shirley ran a different amount each day. She wants to know her daily average for miles she ran during the week. Help her find the answer. Show your work.

Students can create their own logs. Set guidelines to include decimals, number of events to include and other guidelines based on student's experiences with decimals.
Students work with partners or in small groups to create a chart for others to complete. Students might also write questions for others to answer. Students should use gymnastic events, swimming events, track events, or ideas they may have.

Notes and textbook references

Running Log

Day	Mileage	Comments

Find the total miles for the week, . . .for a month.

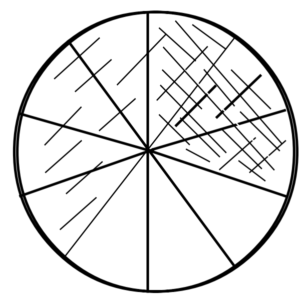
C. Provide students with a blank decimal square or a blank hundred board. Color your first and last initials on the decimal square. Determine the worth of the first initial in decimal form. Next, determine worth of the last initial in decimal form. Finally, add the decimal amounts to tell what part of the square is colored. See Blackline Master I - 27.

D. Have students work in pairs with each pair having a number cube, a different color marker or crayon, and a circle "Tenths" recording sheet (Blackline Master I - 35). Each student in turn will roll the cube, color in on a circle the number of tenths to correspond with the number on the die, and write a fraction to describe that colored part of the circle. (The second student should begin to color on the first circle and continue on a second circle if necessary.) The students add the two fractions and record the sum. (Cubes with different numbers of sides may be used to vary the number of possible fractions.) How many sums were greater than a whole? Less than a whole?

Notes and textbook references

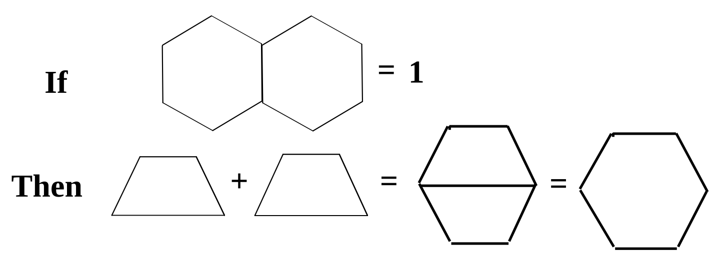
E. Students work in pairs and use the circle “Tenths” recording sheet (Blackline Master I - 35) and two number cubes. Each student rolls the cubes and writes a fraction using the sum of the pips on the cubes as the numerator and 10 as the denominator. If either student rolls a sum of 11 or 12, roll again. The student with the larger fraction colors that part of the circle very lightly. The student with the smaller fraction colors with a darker color his or her fraction, marking off sections that have already been lightly shaded. Together, students record the fractions and the results of subtraction.

$$\frac{7}{10} - \frac{3}{10} = \frac{4}{10}$$



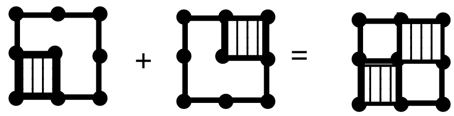
F. Repeat the activities described in 1.04a) - E and 1.04a) - F. Use the “Twelfths” recording sheet (see Blackline Master I - 36).

G. Using pattern blocks, determine what the whole (unit) will be. (This activity may be repeated using the hexagon as the unit or the double hexagon as the unit.) Then record picture and number sentences to illustrate addition and subtraction of fractions with like denominators.

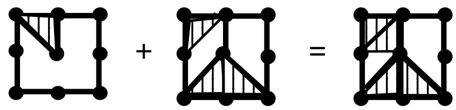


$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

H. Students create wholes on dot paper, shade in parts of two wholes, and record number sentences. See Blackline Master I - 37.



$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$



$$\frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$$

Notes and textbook references

Notes and textbook references

Developing a classroom culture in which mistakes can be made and discussed as sites for learning is worth taking the time to do. When the methods and solutions are the focus rather than the focus being on the children who suggest the method or make the incorrect solution, true mathematical communication and learning can take place. It is our place as teachers to work toward this kind of culture in our classrooms.

b.) Estimate sums and differences.

A. Collect grocery receipts or make up ones of your own. Present them to the students without the totals shown. Have students estimate the total cost of the groceries, then compare their estimates to the actual subtotals (not including the tax). Have the students discuss their strategies for estimating. Have them ask adults in their life how they use estimation in the grocery store.

B. If your school has the children participating in a book order program, have the children estimate the total for their order. You might have them estimate the total for the class after you have the real (or estimated) totals for each child who orders. These kinds of activities give the children a sense of how they will use these skills in the real world.

C. Using metric units, have children measure distances around the school such as the length of a hall or the distance from the classroom door to the water fountain, etc. They should record them in decimal units, such as 6.7 meters. Have them estimate sums and differences between the distances. You might give it a context such as if we walk from the classroom to the water fountain and then back to the classroom and then to the cafeteria, how many meters will we have walked? For smaller units, use distances and lengths in the classroom, such as desks or tables or window widths. Again have the students record measurements in decimal units, such as 14.8 decimeters. Then estimate sums and differences, using questions such as if we line up three desks, about how long will they be? or about how much longer is the counter than this table?

D. Using copies of simple recipes, have the children estimate how much of each ingredient will be needed if the recipe is doubled, tripled, or if you make enough for the whole class.

E. Using a map of North Carolina, have students find distances between given cities. Then have them estimate the distance they would travel if they went from one city to another to another, for example, from Greensboro to High Point to Winston-Salem. Use distances with decimals or fractions or whole numbers, depending on what you want the children to practice. One example: The distance from our town to the next town west of here is 6.7 miles. The distance from our town to the next town east of here is 13.2 miles. Estimate the distance from the east town to the west town. (Asking “about how far” questions should give children the hint to estimate. Often they are uncomfortable with estimations and want a “real” answer.)

F. Close to a Dollar

Notes and textbook references

Students will need a partner to play **Close to a Dollar**. One player will use a calculator. The other player will need pencil and paper. The teacher will need to make up receipts or save receipts from various stores. Parents or other teachers might donate receipts.

How to Play
Below is a receipt from a grocery store. Try to estimate the price on the receipt before your partner can find the total on the calculator. Your total has to be within a dollar. This means you might be over or under by $0.99.

Challenge: After many opportunities playing **Close to a Dollar**, students might play Close to Fifty Cents.

Students can write receipts to use for the game. Receipts should have the store name, the items purchased, and the cost of each item.

Notes and textbook references

G. Trips in North Carolina

Students will need maps with scale of miles, rulers, and a blank travel log. Students work with a partner but each person should create a Travel Log.

Directions:
You and your partner will plan a journey across North Carolina. Students will leave from their hometown and return to their hometown. On your trip you may visit four to six different places but you cannot travel more than 1000 miles. As you plan your trip, you will need to keep a travel log that tells the places you will visit and the distance or mileage you will travel.

Travel Log: My North Carolina Trip (Suggested recording of trip)

Begin	End	Number of Days	Distance

Students should explain how they figured distances between the two places. See Blackline Master I - 28.

Comments:

If students are traveling to another place in North Carolina, students would comment on distance traveled so far and miles left to travel. Student travel logs should document their work.

Extension:
Students might be challenged to create a trip of 800 miles.
Students could create a travel log to somewhere in the United States.

c.) Judge the reasonableness of solutions.

Notes and textbook references

This goal needs to be considered as a matter of course in all the mathematics work the children do. Emphasize the need to judge solutions as to reasonableness as the children work on any of the goals. Children often think they are "finished" once they have a solution, and we must convince them that there is one more step: to look at the solution as compared to the problem or question to see if it makes sense.

A. Present the class with problems and several possible solutions. Have them choose the one that makes the most sense. Have them tell why that one makes sense and why the others do not. Some of these problems can come from the children's own work. Without naming whose solutions you are discussing, the children can discuss why some solutions make sense and others don't.

B. Make up a sheet of problems from the students' errors. Have the students find the errors and discuss why the errors were made. Validate answers by logical reasoning or by using inverse operations. (See Blackline Master I - 42 for an error pattern sheet, Dizzy Division.)

C. For every problem you ask the students to work, focus on verifying results. Give the students ample opportunities to use different strategies to solve a problem. Look for different strategies to solve problems. Ask questions such as these: *"How many whole number digits will be in the answer when you multiply 634 x 45?"* or *"Why would a survey of only 10 fifth graders not be a good way to determine students' favorite pastimes?"*

D. Develop a group of problems that have unreasonable solutions. Conduct classroom or group discussions about why the solutions are not reasonable. Example: The mayor said in her speech that she would not raise taxes. Later, during the same speech, she said she would build a new park for the city and repair the sidewalks and streets. Are her claims reasonable? Why or why not?

E. A technique that focuses students' thinking in a new direction involves looking for numbers that *couldn't* be the answer to a given problem! After stating the problem ask students to suggest numbers that can't be the solution. List them on the blackboard and have other students explain why those numbers won't do the trick. In analyzing the problem from this angle, students explore the limits and sense of magnitude that the solution requires. Everyone can give a *wrong* answer and the only student who *is wrong* is the one who gives the correct solution!

Notes and textbook references

F. Assign a few problems from your text or other source and have the students write one wrong answer for each problem (the wrong answers should reflect errors that a fifth grader would make). Students should be prepared to explain why the answer is incorrect.

G. Play **You Be the Judge.** Copy Blackline Master I - 43 on cardstock, laminate, and cut cards apart. Draw a two column chart on the board with two headings: " Object!" "Sustain!"

Directions for the game**:**

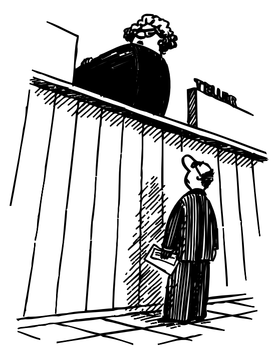
- Students must decide whether or not the math on the cards was done correctly (Sustain!) or incorrectly (Object!). Allow time for students to "defend" their decisions. Teacher holds cards up one at a time for the students to discuss the reasonableness of solutions and place them in the proper columns.

- After all cards have been placed, the teacher gives the students one last opportunity to make changes. Ask if they rest their case and are ready for a verdict.

- If all cards are place properly the students win their case.

- Note: Additional problems can be written from common mistakes your students make in class and black graduation gowns make wonderful robes for judges and will increase the students' motivation.

Variation: Students could be divided into three groups - lawyers (defense and plaintiff) and a jury. Assign appropriate rolls. Lawyers receive problems to defend the next day. Jury gives verdict. Roles can be rotated.



H. After playing the game **You Be the Judge** 1.04c) G, have students write distracters for the problems and rewrite it as a multiple choice item. Form groups to discuss possible distracters for these problems and why it is important to look at each answer choice. Have students brainstorm strategies they can/should do to check for the reasonableness of solutions on all problems – multiple choice or not.

Notes and textbook references

Notes and textbook references

1.05 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

A. **Structure of Story Problems** The structure of problems fosters understanding of the relationship of multiplication and division. Make cubes and other manipulatives available.

Write problems based on interests of students, classroom events, the school and community environment as well as content in other areas of the curriculum. Structure problems similar to problems below:

Blackline Master III - 8 has a problem-solving guide that may be helpful to students.

Cindy works in a candy factory and is packaging candy in boxes. She is able to arrange 24 candies in one layer. If a box holds 96 pieces of candy, how many layers of candy can fit into one box?

Cindy works in a candy factory and is packaging candy in boxes. By making 4 equal layers, Cindy is able to put 96 candies in each box. How many candies are in each layer?

Cindy works in a candy factory and is packaging candy in boxes. She arranged the candy in 4 layers with 24 pieces of candy in each layer. How many pieces of candy did Cindy put in each box?

Mark and Tim are working in a candy factory. Mark was able to fit 14 candies in his box. Tim was able to fit 4 times as many candies in his box. How many candies did Tim's box hold?

B. Use a hundred board (Blackline Masters I - 38 and I - 39) to add any three numbers that are adjacent horizontally, vertically, or diagonally. Look for patterns in the sums of the three numbers. (Students should note that all sums are divisible by three.) Ask students whether the pattern will apply whether they add five adjacent numbers. Would this be the same for numbers on the calendar?

Notes and textbook references

C. Use a calculator to find the products of the following equations. Look for a pattern in the products.

11 x 9 =
22 x 9 =
33 x 9 =
44 x 9 =
Using only the pattern, answer the problems:
66 x 9
77 x 9
What do you think would happen if you multiplied:
11 x 99
22 x 99
33 x 99
Check with your calculator.

D. Read <u>The King's Chessboard</u> by David Birch. Use a calculator to find out how many grains of rice the king must send to the peasant if he gives him one grain on the first day and doubles the amount each day.

E. Choose a number between 2 and 12. Key in the number you have chosen. Press [+] [=] . Continue to press the [=] key. Write down the number you get each time you punch [=] . Push [=] at least 10-12 times, then discuss the pattern.

F. Find out the number of seats in the Dean Dome in Chapel Hill. Can you find three N.C. towns whose entire population could go to a concert together?

G. Have students make up their own word problems leaving out some important information. Ask them to present their problem to the class and have the group discuss what is needed in order to work with them successfully.

Notes and textbook references

H. Give students a variety of problems, with and without sufficient data, asking them for the solution. If they decide that there is insufficient information, ask them to supply the missing information.

I. Have the students list the information necessary to find the answers to questions such as:

1. How much change would you get back from a lunch order at McDonald's?
2. How many bricks would be needed to build a house?
3. How many drops of water would you need to fill a swimming pool?
4. How much would it cost a family to get into Disney World?

J. Give students a series of problems to work. Then give them clues to eliminate answers until they have one left to give them the information they need. For example:

632	260	911	573	165
-616	-18	- 819	-546	-39
87	900	54	324	593
-28	-23	- 26	- 312	-429

How long did the Wright brothers stay in the air on their first flight? The number of minutes was not:

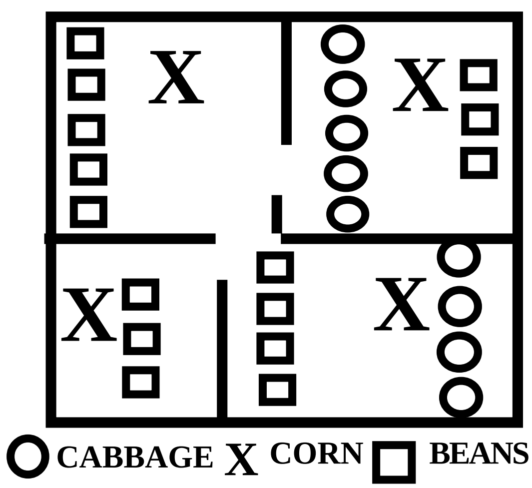
a.	8 x 8	e.	a palindromic number
b.	10 more than 82	f.	more than 8 hundred
c.	2 sets of 10 plus 8	g.	12 tens and 4 ones
d.	5 tens and 9 ones	h.	4 x 4 or 3 x 9

Notes and textbook references

K. Work with other faculty members to establish a problem-solving shelf in your professional resources area. Good sources of ideas are the sample books sent for adoption review, Math Stars, *SuperStars* (available through LEA central offices, DPI Publications Office 1-800-663-1250 or the Mathematics website: http://community.learnnc.org/dpi/math), and magazines such as *Zillions* and *Dynamath*.

L. The Wright brothers first flew in North Carolina. Interview 10 adults. How many of them have flown? How would you write this as a fraction? What fraction of this group has not flown?

M. Whose garden is whose? Susan, Al, John, and Tonya have gardens. Figure out whose garden is whose by using these clues:
The boys both have an odd number of plants.
One of the girls has the smallest garden .
John has no cabbage.
Susan has the same number of each crop.



Encourage students to write logic puzzles for their friends.

N. Read <u>*George Washington's Breakfast*</u> by Jean Fritz. What did George Washington have for breakfast? How did Washington determine the number of seeds in a pound of red clover or a pound of Red River grass?

Notes and textbook references

O. Provide a copy of the chart and a calculator to students. Then ask them questions similar to those below the chart.

Mountain Peaks of North Carolina

Mountain	Height (feet)
Grandfather Mountain	5,964
Mt. Mitchell	6,684
Mt. Sterling	5,835
Mt. Hardison	6,134

1. How much taller is Mt. Mitchell than Mt. Sterling?
2. If you climbed Mt. Hardison and Grandfather Mountain, how many total feet would you have climbed?
3. List the mountains you climbed if you climbed 29,732 feet in all.
4. List the mountains you climbed if you climbed 11,959 feet in all.

Encourage students to create their own questions.

P. Placing students into small groups, assign a specific task, asking the group to generate two or three different ways to solve the problem.

Q. Play a version of "**Guess My Number**" using hundred boards. The children may ask five "yes or no" questions to determine the mystery number. Stress the importance of asking relevant questions to narrow down the choices. A good question might be: Is the number greater than or equal to 50 ? A poor beginning question might be: Is the number on the first row? Encourage children creatively to tie in questions concerning even and odd numbers, multiples, factors, sum of digits, etc. What is the least number of questions needed to be absolutely certain of the answer? See blackline Masters I - 38 or I - 39.

Notes and textbook references

R. **Free Ride**: Place 10 toothpicks on the table. You and your partner take turns picking up either one or two toothpicks. The player to pick up the last toothpick wins a free ride. Can you figure out how to get a free ride every time? Help students look for a pattern by:

- working backwards
- making a simpler problem
- making a table to illustrate the pattern

S. Draw a circle on a map of the southeastern United States with the center at Murphy and the radius of the circle reading Raleigh. (Your circle's diameter will be 710 miles since it is 355 miles from Raleigh to Murphy.) Ask students to explore:

- Which state capitals are closer to Murphy than to Raleigh?
- Which is closer to Murphy - Atlanta, Georgia or Charlotte?
- It is more than 500 miles from Murphy to Manteo. Suppose you traveled 500 miles from Murphy in other directions. Where might you go?

T. Using their social studies book and other sources, have students find and compute the area of the three largest counties, the area of the three smallest counties, the population of the three largest counties, etc.

U. Have students collect the last four digits of the telephone numbers of 10 friends. Have them subtract these numbers from 9,105. Is there anyone whose number cannot be subtracted?

V. Have a "problem of the day" board and ask students to write down their strategies and solutions with extra credit for trying and solving hard questions.

Notes and textbook references

W. The students brainstorm ways to describe their math books in terms of numbers: its width, the number of pages, the publication date, a student -generated “quality rating” of the book, the area of the cover, and so on.

X. Have students write problems that are especially suited to each of the strategies.